Empirical formulae of the masses of elementary particles

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particle	formula	calculated(c)	measured(m)	$\lvert c/m-1\rvert$
e	$1/(12\pi^2)\epsilon_0^{1/3}(1+(1/4)(1/(6\pi)^2))^{-1}M_{pl}$	0.511002 MeV	$0.510998946 \pm 0.0000000031$ MeV	5.9×10^{-6}
μ	$3/2\epsilon_0^{1/3}(1-1/(2\pi)+3/(4\pi)^2)^{-1}M_{pl}$	105.6594 MeV	$105.6583745 \pm 0.0000024$ MeV	9.6×10^{-6}
τ	$9\pi\epsilon_0^{1/3}(1-1/(8\pi)+(5/4)(1/(6\pi)^2))^{-1}M_{pl}$	1.77684 GeV	1.77686 ± 0.12 GeV	1.9×10^{-5}
t	$8\times(6\pi)^2\epsilon_0^{1/3}M_{pl}$	172.1 GeV	173.4 ± 0.75 GeV	7.5×10^{-3}
c	$12\epsilon_0^{1/3}M_{pl}$	1.24 GeV	1.28 ± 0.025 GeV($\overline{\mathrm{MS}}$ at m_c)	3.4×10^{-2}
u	$8\times(6\pi)^{-2}\epsilon_0^{1/3}M_{pl}$	2.07 MeV	2.15 ± 0.15 MeV ($\overline{\mathrm{MS}}$ at 2 GeV)	3.8×10^{-2}
b	$3\times2^{-1/12}(6\pi)\epsilon_0^{1/3}M_{pl}$	4.33 GeV	4.18 ± 0.03 GeV($\overline{\mathrm{MS}}$ at m_b)	3.6×10^{-2}
s	$\epsilon_0^{1/3}M_{pl}$	91.8 MeV	93.8 ± 2.4 MeV ($\overline{\mathrm{MS}}$ at 2 GeV)	2.1×10^{-2}
d	$(6\pi)^{-1}\epsilon_0^{1/3}M_{pl}$	4.87 MeV	4.7 ± 0.2 MeV ($\overline{\mathrm{MS}}$ at 2 GeV)	3.6×10^{-2}
Z	$1/(8\pi^2)\epsilon_0^{1/4}(1+(1/11)(1/(2\pi)^2))^{-1}M_{pl}$	91.1883 GeV	91.1876 ± 0.0021 GeV	7.4×10^{-6}
W	$2^{-1/4}/(8\pi^2)\epsilon_0^{1/4}(1-(19/11)(1/(2\pi)^2))^{-1}M_{pl}$	80.373 GeV	80.385 ± 0.015 GeV	1.9×10^{-4}
H	$2^{1/2}/(8\pi^2)\epsilon_0^{1/4}(1+(14/11)(1/(2\pi)^2))^{-1}M_{pl}$	125.22 GeV	125.1 ± 0.2 GeV	1.0×10^{-3}

I report emprical formulae of the masses of charged leptons (e,μ,τ), quarks (t,c,u,b,s,d), gauge bosons (Z,W), and Higgs boson (H). The formulae yield the masses in terms of the Planck mass M_{pl} and a dimensionless constant $\epsilon_0 = 2\times(6\pi)^{-48}$. There is no adjustable parameter in the formulae.

The mass values calculated using the formulae are compared with measured values. For the calculation, the value of Planck mass from CODATA[1)] is used:

$$M_{pl} = 1.220910 \pm 0.000029 \times 10^{19}\ \mathrm{GeV}.$$

The measured values of particle masses are taken from PDG2016.[2)] The mass of a quark is dependent on the scale and the scheme. In the PDG review, the mass of quarks other than the t quark are given in the $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV for the u, d, s quarks and at $\mu = m_q$ for the b and c quarks. For the t quark, the measured mass is considered to be the pole mass. The formula for a quark is assumed to yield the $\overline{\mathrm{MS}}$ mass at the Z boson mass m_Z. The first-order renormalized group equation (RGE) below is used to correct for the mass value at m_Z to the mass at the scale the PDG uses:

$$\frac{m(t)}{m_0} = \exp\left(-\int \frac{\alpha_s(t)}{\pi}dt\right),$$

where $t = \log\mu^2$ and $\alpha_s(t)$ is the running QCD coupling constant at the scale μ. The value of $\alpha_s(t)$ in the PDG2016 review is used for the calculation.

The formulae, the calculated values (c) using the formulae, the measured values (m), and difference $|c/m-1|$ are summarized in the table above. The calculation reproduce the measured mass values well. The agreement is within the uncertainty of the measured mass or the Planck mass (2.4×10^{-5}).

There is a pattern in the formulae. The formulae can be summarized as

$$m_l = \frac{1}{2}N_l(6\pi)^{n_l}\epsilon_0^{1/3}(1+\delta_l)^{-1}M_{pl},$$
$$m_q = 2^{c_q}N_q(6\pi)^{n_q}\epsilon_0^{1/3}M_{pl},$$
$$m_B = \frac{2^{c_B}}{8\pi^2}\epsilon_0^{1/4}(1+\delta_B)^{-1}M_{pl},$$

where m_l, m_q, and m_B are the masses of charged leptons, quarks, and bosons, respectively. N_l and N_q are small positive integers, n_l and n_q are integers ranging from -2 to 2, and δ_l and δ_B are small real numbers. $c_b = -1/12$ and $c_q = 0$ for all other quarks, and $c_Z = 0$, $c_W = -1/4$, and $c_H = 1/2$. The pattern suggests the existence of a rule that determines the formulae. Note that all fermion masses are order of $(6\pi)^{n_f}\epsilon_0^{1/3}$ with $-2 \le n_f \le 2$ and the boson masses are of the order of $1/8\pi^2\epsilon_0^{1/4}$. Presumaby, this part is the main part of the mass, and $(1+\delta_l)$ and $(1+\delta_B)$ are correction factors due to interactions.

Note that the value of ϵ_0 is consistent with the product of the Hubble constant H_0 and the Planck time $t_{pl} = 1/M_{pl}$:

$$H_0 \times t_{pl} = (1.211 \pm 0.014)\times10^{-61}$$
$$\epsilon_0 \equiv 2\times(6\pi)^{-48} = 1.220608\times10^{-61}$$

Here, the WMAP 9 year value of H_0 is used. This suggests that the masses of elementary particles are related to the expansion of space-time.

A theoretical model that can explain these formulae is under development.

References

1) Rev. Mod. Phys. **88**, 053009 (2016).
2) Review of Particle Physics, Chin. Phys. C **40**, 100001 (2016).

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