Updated empirical formulae of the masses of elementary particles

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	formula	calculated(c)	measured(m)	$\lvert c/m-1\rvert$
e	$1/(12\pi^2)\epsilon_0^{1/3}(1+(1/4)(1/(6\pi)^2))^{-1}M_{pl}$	0.511002 MeV	$0.510998946 \pm 0.0000000031$ MeV	5.9×10^{-6}
μ	$3/2\epsilon_0^{1/3}(1-1/(2\pi)+3/(4\pi)^2)^{-1}M_{pl}$	105.6594 MeV	$105.6583745 \pm 0.0000024$ MeV	9.6×10^{-6}
τ	$9\pi\epsilon_0^{1/3}(1-1/(8\pi)+(5/4)(1/(6\pi)^2))^{-1}M_{pl}$	1.77684 GeV	1.77686 ± 0.12 GeV	1.3×10^{-5}
t	$8 \times (6\pi)^2\epsilon_0^{1/3}M_{pl}$	172.1 GeV	173.4 ± 0.75 GeV	7.5×10^{-3}
c	$12\epsilon_0^{1/3}M_{pl}$	1.24 GeV	1.28 ± 0.025 GeV($\overline{\text{MS}}$ at m_c)	3.4×10^{-2}
u	$8 \times (6\pi)^{-2}\epsilon_0^{1/3}M_{pl}$	2.07 MeV	2.15 ± 0.15 MeV ($\overline{\text{MS}}$ at 2 GeV)	3.8×10^{-2}
b	$3 \times (6\pi)\epsilon_0^{1/3}(1+1/4\pi+3/(4\pi)^2)^{-1}M_{pl}$	4.18 GeV	4.18 ± 0.03 GeV($\overline{\text{MS}}$ at m_b)	$\leq 2 \times 10^{-3}$
s	$\epsilon_0^{1/3}M_{pl}$	91.8 MeV	93.8 ± 2.4 MeV ($\overline{\text{MS}}$ at 2 GeV)	2.1×10^{-2}
d	$(6\pi)^{-1}\epsilon_0^{1/3}(1+1/6\pi)^{-1}M_{pl}$	4.63 MeV	4.7 ± 0.2 MeV ($\overline{\text{MS}}$ at 2 GeV)	1.5×10^{-2}
Z	$1/(8\pi^2)\epsilon_0^{1/4}(1+1/12(1/(6\pi))+1/12(1/(6\pi)^2))^{-1/2}M_{pl}$	91.1862 GeV	91.1876 ± 0.0021 GeV	1.5×10^{-5}
W	$2^{-1/4}/(8\pi^2)\epsilon_0^{1/4}(1-1/(4\pi)-2/(4\pi)^2)^{-1/2}M_{pl}$	80.387 GeV	80.385 ± 0.015 GeV	2.5×10^{-5}
H	$2^{1/2}/(8\pi^2)\epsilon_0^{1/4}(1+1/(4\pi)+1/(4\pi)^2)^{-1/2}M_{pl}$	125.14 GeV	125.1 ± 0.2 GeV	3.8×10^{-4}

In my article in APR2017[1)] I reported empirical formulae of the masses of charged leptons (e, μ, τ), quarks (t, c, u, b, s, d), gauge bosons (Z, W), and Higgs boson (H). The formulae yield the masses in terms of the Planck mass M_{pl} and a dimensionless constant $\epsilon_0 = 2 \times (6\pi)^{-48}$. There is no adjustable parameter in the formulae. Note that the value of ϵ_0 is consistent with the product of the Hubble constant H_0 and the Planck time $t_{pl} = 1/M_{pl}$:

$$H_0 \times t_{pl} = (1.211 \pm 0.014) \times 10^{-61}\,,$$
$$\epsilon_0 \equiv 2 \times (6\pi)^{-48} = 1.220608 \times 10^{-61}\,.$$

Here, the WMAP nine-year value of H_0 is used. This suggests that the masses of elementary particles are related to the expansion of space-time.

Here, I report a small update of the formulae for b and d quarks and Z, W, H bosons. The revised formulae have better agreement with the data.

The mass values calculated using the formulae are compared with the measured ones. For the calculation, the value of Planck mass from CODATA[2)] is used:

$$M_{pl} = 1.220910 \pm 0.000029 \times 10^{19}\ \text{GeV}.$$

The measured values of the particle masses are taken from PDG2016.[3)] In QCD, the mass of a quark is dependent on the scale and the scheme. In the PDG review, the mass of quarks other than the t quark are given in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV for the u, d, s quarks and at $\mu = m_q$ for the b and c quarks. For the t quark, the measured mass is considered to be the pole mass. The formula for a quark is assumed to yield the $\overline{\text{MS}}$ mass at the Z boson mass m_Z. The first-order renormalization group equation (RGE) given below is used to correct for the mass value at the scale m_Z to the mass at the scale the PDG uses:

$$\frac{m(t)}{m_0} = \exp\left(-\int \frac{\alpha_s(t)}{\pi}dt\right),$$

where $t = \log\mu^2$ and $\alpha_s(t)$ is the running QCD coupling constant at the scale μ. The value of $\alpha_s(t)$ in the PDG2016 review is used for the calculation.

The formulae, calculated values (c) using the formulae, measured values (m), and difference $|c/m-1|$ are summarized in the table above. The calculations reproduce the measured mass values well.

The agreement is within the uncertainty of the measured mass or the Planck mass (2.4×10^{-5}). There is a pattern in the formulae. The formulae can be summarized as

$$m_f = \frac{1}{2}N_f(6\pi)^{n_f}\epsilon_0^{1/3}(1+\delta_l)^{-1}M_{pl},$$
$$m_B = \frac{2^{c_B}}{8\pi^2}\epsilon_0^{1/4}(1+\delta_B)^{-1}M_{pl},$$

where m_f and m_B are the masses of charged fermions (leptons and quarks) and bosons, respectively. N_f are small positive integers, n_f are integers ranging from -2 to 2, and δ_l and δ_B are small real numbers. $c_Z = 0$, $c_W = -1/4$, and $c_H = 1/2$. The pattern suggests the existence of a rule that determines the formulae. Note that all fermion masses are of order of $(6\pi)^{n_f}\epsilon_0^{1/3}$ with $-2 \leq n_f \leq 2$ and the boson masses are of the order of $1/8\pi^2\epsilon_0^{1/4}$. Presumaby, this part is the main part of the mass, and $(1+\delta_l)$ and $(1+\delta_B)$ are the correction factors due to interactions.

A model to explain these formulae is reported in a separate article.[4)]

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References
1) Y. Akiba, RIKEN Accel. Prog. Rep. **51** (2017).
2) P. J. Mohr *et al.*, Rev. Mod. Phys. **88**, 035009 (2016).
3) C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
4) Y. Akiba, RIKEN Accel. Prog. Rep. **52** (2018).