Empirical formulas for the standard-model parameters

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In my previous article,[1)] I reported empirical formulas of the masses of the elementary particles in the standard model (SM), namely, charged leptons (e, μ, τ), quarks (t, c, u, b, s, d), gauge bosons (Z, W), and the Higgs boson (H). Each of these formulas yields the ratio μ_p/M_{pl} of the mass of particle p to the Planck mass $M_{pl} = 1.220910 \pm 0.000029 \times 10^{19}$ GeV in terms of a dimensionless constant $\epsilon_0 = 2 \times (6\pi)^{-48}$. There is no adjustable parameter in the formulas.

Here, I report similar formulas for the mass of neutrinos, Cabbibo-Kobayashi-Masukawa (CKM) quark mixing parameters, and neutrino mixing parameters. Table 1 lists mass formulas including neutrions. The neutrino masses calculated from the formulas are $m_1 = 2.70 \times 10^{-3}$ eV, $m_2 = 9.01 \times 10^{-3}$ eV, and $m_3 = 5.09 \times 10^{-2}$ eV. Table 2 shows the formulas for the CKM matrix elements and their calculated values. Table 3 lists the formulas of the neutrino mixing parameters and their calculated values. The values calculated from these formulas all agree with experimental data within the uncertainty of the data.

Table 1. Formulas for the masses of the SM particles.

particle p	formula ($\mu_p = m_p/M_{\rm pl}$)
e	$\frac{1}{12\pi^2}\epsilon_0^{1/3}\left(1+\frac{1}{4}\frac{1}{(6\pi)^2}\right)^{-1}$
μ	$\frac{3}{2}\epsilon_0^{1/3}\left(1-\frac{3}{6\pi}+\frac{27}{4}\frac{1}{(6\pi)^2}\right)^{-1}$
τ	$9\pi\epsilon_0^{1/3}\left(1-\frac{3}{4}\frac{1}{6\pi}+\frac{5}{4}\frac{1}{(6\pi)^2}\right)^{-1}$
ν_1	$\frac{2}{3}\epsilon_0^{1/2}\left(1+\frac{1}{6\pi}\right)^{-1}$
ν_2	$2\epsilon_0^{1/2}\left(1-\frac{1}{6\pi}\right)^{-1}$
ν_3	$4\pi\epsilon_0^{1/2}\left(1+\frac{1}{6\pi}\right)^{-1}$
t	$8(6\pi)^2\epsilon_0^{1/3}$
c	$12\epsilon_0^{1/3}$
u	$8(6\pi)^{-2}\epsilon_0^{1/3}$
b	$3(6\pi)\epsilon_0^{1/3}\left(1+\frac{3}{2}\frac{1}{6\pi}+\frac{27}{4}\frac{1}{(6\pi)^2}\right)^{-1}$
s	$\epsilon_0^{1/3}$
d	$(6\pi)^{-1}\epsilon_0^{1/3}\left(1+\frac{1}{6\pi}\right)^{-1}$
Z	$\frac{1}{(8\pi^2)}\epsilon_0^{1/4}\left(1+\frac{1}{12}\frac{1}{6\pi}+\frac{1}{12}\frac{1}{(6\pi)^2}\right)^{-1/2}$
W	$\frac{2^{-1/4}}{(8\pi^2}\epsilon_0^{1/4}\left(1-\frac{3}{2}\frac{1}{6\pi}-\frac{9}{4}\frac{1}{(6\pi)^2}\right)^{-1/2}$
H	$\frac{2^{1/2}}{8\pi^2}\epsilon_0^{1/4}\left(1+\frac{3}{2}\frac{1}{6\pi}-\frac{9}{2}\frac{1}{(6\pi)^2}\right)^{-1/2}$

Table 2. Formulas of the CKM matrix elements V_{us}, V_{cb}, V_{ub}, and the CP-violation parameter $\bar{\eta}$.

	formula	calculated
V_{us}	$\left(\frac{1}{6\pi}\left(1+\frac{1}{6\pi}\right)^{-1}\right)^{1/2}$	0.22445
V_{cb}	$\left(\frac{2}{3}\right)^{1/2}\frac{1}{6\pi}$	0.04332
V_{ub}	$\frac{4}{3}\frac{1}{(6\pi)^2}$	0.003753
$\bar{\eta}$	$\left(1+\frac{3}{2}\frac{1}{6\pi}+\frac{27}{4}\frac{1}{(6\pi)^2}\right)\frac{1}{\pi}$	0.3497

Table 3. Formulas of the neutrino-mixing matrix.

	formula	calculated
s_{12}	$\left(\frac{1}{3}\left(1-\frac{1}{6\pi}\right)\left(1+\frac{1}{6\pi}\right)^{-1}\right)^{1/2}$	0.547
s_{23}	$\left(\frac{3}{2\pi}\left(1+\frac{1}{6\pi}\right)\left(1-\frac{1}{6\pi}\right)^{-1}\right)^{1/2}$	0.729
s_{13}	$\left(\frac{1}{12\pi}\right)^{1/2}\left(1-\frac{1}{6\pi}\right)\left(1+\frac{1}{6\pi}\right)^{-1}$	0.146

There are 25 free parameters in the SM. These formulas yield 22 out of the 25 parameters. The remaining 3 parameters are the fine structure constant α, the strong coupling constant α_s, and the neutrino CP violation parameter δ_{CP}. Both α and α_s are scale dependent, and δ_{CP} is the only unmeasured parameter in the SM.

Note that the value of ϵ_0 is consistent with the product of the Hubble constant H_0 and the Planck time $t_{pl} = 1/M_{pl}$:

$$H_0 \times t_{pl} = (1.211 \pm 0.014) \times 10^{-61}.$$
$$\epsilon_0 \equiv 2 \times (6\pi)^{-48} = 1.220608 \times 10^{-61}.$$

Here, the Wilkinson Microwave Anisotropy Prob (WMAP) nine-year value of H_0 is used. This suggests that the masses of elementary particles are related to the expansion of spacetime.

A model to explain these formulae is reported in the next article,[2)] and implications to gravity and cosmology are reported in the article appearing after that.[3)]

References

1) Y. Akiba, Accel. Prog. Rep. **52**, 98 (2019).
2) Y. Akiba, in this report.
3) Y. Akiba, in this report.

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