Quarternion-spin-isospin model for the standard-model parameters

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In the preceding article,[1] empirical formulas of the parameters of the standard model (SM) of particle physics are presented. Here, I report a model that can produce these formulas. We call the model the "Quarternion-spin-isospin model" since it is based on operators that are products of the quarternion bases I^μ, spin operator σ^ν, and (weak) isospin operator τ^a. In the model, the Planck time $\tau_{pl} = 1/M_{pl} = 5.3912 \times 10^{-44}$ s is the minimum duration of time. A term of the Lagrangian density $\mathcal{L}_i$ of a particle is an "oriented product" of 48 "normalized primordial actions" (NPAs) that are selected from the following 64 NPAs.

$$\left\{\frac{I^\mu \sigma^\nu \tau^a}{6\pi}, \frac{i}{3\pi}, \frac{\tau^3}{3\pi}, \frac{I^c\tau^3}{3\pi}, \frac{-I^c}{2\pi}, \frac{-i}{\pi}, i, I^c, -2, 2, 1\right\}$$

The oriented product operator $\vee$ has the following reduction rules:

$$\hat{\alpha} \vee \hat{\beta} = \begin{cases} \hat{\alpha}\hat{\beta}, & (\text{if } \hat{\alpha}\hat{\beta} = -\hat{\beta}\hat{\alpha}) \\ 0, & (\text{if } \hat{\alpha}\hat{\beta} = \hat{\beta}\hat{\alpha}) \end{cases}$$

$$\hat{\alpha}\vee\hat{\beta}\vee\hat{\gamma} = \begin{cases} \hat{\alpha}\hat{\beta}\hat{\gamma}, & (\text{if } \hat{\alpha}\hat{\beta}=-\hat{\beta}\hat{\alpha}, \hat{\beta}\hat{\gamma}=-\hat{\gamma}\hat{\beta}, \hat{\gamma}\hat{\alpha}=-\hat{\alpha}\hat{\gamma}) \\ 0, & (\text{otherwise}) \end{cases}$$

$$s \vee \hat{\alpha} = s\hat{\alpha},\ s_1 \vee s_2 = 0,\ \hat{\alpha} \vee s \vee \hat{\beta} = s\hat{\alpha}\hat{\beta}.$$

Here, s is a scalar. Following these rules, a 48 $\vee$ product of NPAs, $dS = \hat{s}_{i_1} \vee \cdots \vee \hat{s}_{i_{48}}$, is reduced to a value in the form of $\pm m(6\pi)^n i^s (\tau^3)^{s'} \epsilon_0$, where $\epsilon_0 = 2 \times (6\pi)^{-48}$, $s \in \{0,1\}$, $s' \in \{0,1\}$, $m \in \{1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm18, \pm24\}$, and $n \in \{0,1,2\}$. The values of s, s', n, and m are determined by the selection of the subset $\{\hat{s}_1, \cdots, \hat{s}_{48}\}$ from S_{NPA}. Due to the calculation rules of the $\vee$ product, only a limited number of 48 products have a non-zero reduced value. We call these non-zero 48 product the "elementary action" (EA). An EA corresponds to a term of Lagrangian density $\mathcal{L}$ of an elementary particle, *e.g.*, the electron.

We found 48 EAs that correspond to the elementary particles of the SM, which are summarized in Table 1. In the table, U and D denote U-type and D-type quarks, respectively, when their masses are ignored.

The mass of particles can be obtained from Table I. In the following, I show how the mass formula of the electron can be obtained from the table as an example.

The Lagrangian of the electron is a sum of three EAs.

$$\hat{\mathcal{L}}_e = 4(6\pi)^2 i\epsilon_0\tau^3 + i\epsilon_0 i\tau^3 + 3\epsilon_0.$$

Each EA can then be written as a product of the following "operators."

Table 1. Elementary actions.

	$(6\pi)^2\epsilon_0$		$(6\pi)\epsilon_0$				ϵ_0		
	$i\tau^3$	i	$i\tau^3$	i	τ^3	1	$i\tau^3$	i	1
g_{3D}									3
g_{2D}									2
G^{ab}							−24		
e	4						1		3
μ	4		−12				27		3
τ	4		−3		3		1+4		
ν_1		12		12					2
ν_2		−4	4						2
ν_3		12		12	−2	2			
U	−12						2		
u	4								8
c	4								24
t	4								8
D	−12						1		
d	−12		−12		3				
s	−12								3
b	4		6		3		27	18	
Z		12	1				1	18	
H	−4		6					18	
W		12		−18			−27	18	

$$\hat{\partial}_{16} = \epsilon_0^{1/3}(1 + i\sigma^1 + i\sigma^2 + i\sigma^3),$$
$$\hat{\psi}_{15}^{+} = |\psi_{15}|\tau^{+}(u_1^{+}\sigma^1 + u_2^{+}\sigma^2 + u_3^{+}\sigma^3),$$
$$\hat{\psi}_{15}^{-} = |\psi_{15}|\tau^{-}(u_1^{-}\sigma^1 + u_2^{-}\sigma^2 + u_3^{-}\sigma^3),$$
$$\hat{A}_{18} = (6\pi)^{-2}\epsilon_0^{1/3}(1 + i\sigma^1 + i\sigma^2 + i\sigma^3).$$

Here, $|\psi_{15}| = (6\pi)\epsilon_0^{1/3}$ and the 3D vectors $u^+ = (u_1^+, u_2^+, u_3^+)$ and $u^- = (u_1^-, u_2^-, u_3^-)$ satisfy $u^+ \times u^- = (-1,-1,-1)$ and $u^+ \cdot u^- = 1$. The operators $\hat{\partial}_{16}$ and $\hat{\psi}_{15}^-$ correspond to the differential operator $i\sigma^\mu\partial_\mu$ and the electron field operator, and ψ_{15}^+ corresponds to its conjugate. One can show

$$\hat{\mathcal{L}}_e = \left(1 + \frac{1}{4}\frac{1}{(6\pi)^2}\right)\left(\hat{\partial}_{16}\hat{\psi}_{15}^{+}\hat{\psi}_{15}^{-} + \mu_e|\psi_{15}|^2\right),$$
$$\mu_e = \left(1 + \frac{1}{4}\frac{1}{(6\pi)^2}\right)^{-1}\frac{1}{12\pi^2}\epsilon_0^{1/3}.$$

Similarly, all of the 22 formulas of the SM parameters presented in the preceding article are derived. The model also predicts 100% CP violation in the neutrino sector and that the product of the Hubble constant H_0 and the Planck time t_{pl} is $H_0 t_{pl} = 2 \times (6\pi)^{-48}$.

Reference

1) Y. Akiba, in this report.

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