Double charge-exchange phonon states†

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The possibility of inducing double charge-exchange (DCX) excitations by means of heavy-ion beams at intermediate energies has recently fostered interest in new collective excitations such as double isobaric analog states (DIAS) and double Gamow-Teller giant resonances (DGTR). A research program based on a new reaction, (^{12}C, ^{12}Be(0_2^+)), is planned at the RIKEN RIBF facility with high-intensity heavy-ion beams at the optimal energy of $E_{\rm lab}$ = 250 MeV/nucleon to excite the spin-isospin response.[1] A big advantage of this reaction is based on the fact that it is a $(2p, 2n)$-type DCX reaction, and one can use a neutron-rich target to excite DGTR strength. In this report, we present some formulas to evaluate different combinations of the average excitation energies of DIAS and DGTR by using commutator relations for the double isospin $\sum_{i,j=1}^{A} t_-(i)t_-(j)$ and spin-isospin operator $\sum_{i,j=1}^{A} \boldsymbol{\sigma}(i)t_-(i)\boldsymbol{\sigma}(j)t_-(j)$. Here, $\boldsymbol{t} = \boldsymbol{\tau}/2$, and $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ denote the Pauli matrices in spin and isospin space, respectively. Specifically, we present formulas to estimate $E_{\rm DIAS} - 2E_{\rm IAS}$ from the most relevant isospin symmetry breaking (ISB) terms in the nuclear Hamiltonian and $E_{\rm DGTR} - E_{\rm DIAS} - 2(E_{\rm GTR} - E_{\rm IAS})$ from a simple albeit realistic Hamiltonian including separable residual interactions.

The expectation value for the energy of the DIAS is defined as

$$E_{\rm DIAS} \equiv \langle {\rm DIAS}|\mathcal{H}|{\rm DIAS}\rangle - \langle 0|\mathcal{H}|0\rangle \ , \qquad (1)$$

where $|0\rangle$ represents the ground state, and

$$|{\rm DIAS}\rangle \equiv \frac{T_-|{\rm IAS}\rangle}{\langle {\rm IAS}|T_+T_-|{\rm IAS}\rangle^{1/2}} \qquad (2)$$

is the definition of the DIAS state in terms of the IAS that, in turn, can be written as $|{\rm IAS}\rangle \equiv T_-|0\rangle/\langle 0|T_+T_-|0\rangle^{1/2}$, where $T_+ = \sum_i^A t_+(i)$ and $T_- = \sum_i^A t_-(i)$ are the isospin raising and lowering operators, respectively.

Starting from Eq. (1) and the definitions of DIAS and IAS previously given, one may write the excitation energy of DIAS as

$$E_{\rm DIAS} = \frac{\langle 0|[T_+^2, [\mathcal{H}, T_-^2]]|0\rangle}{\langle 0|T_+^2 T_-^2|0\rangle} \ , \qquad (3)$$

assuming that the ground state has good isospin; in other words, there is no isospin mixing, and $T_+|0\rangle = 0$. Remembering that the $E_{\rm IAS}$ is written as, within the same approximation (*i.e.*, no isospin mixing in the ground state), $E_{\rm IAS} = \langle {\rm IAS}|\mathcal{H}|{\rm IAS}\rangle - \langle 0|\mathcal{H}|0\rangle = \langle 0|[T_+, [\mathcal{H}, T_-]]|0\rangle/\langle 0|T_+T_-|0\rangle$, one can eventually write

$$E_{\rm DIAS} = 2E_{\rm IAS} + \frac{\langle 0|[T_+, [T_+, [[\mathcal{H}, T_-], T_-]]]|0\rangle}{2(N-Z)(N-Z-1)}. \qquad (4)$$

The energy of the double GT state can be defined in an analogous way to Eq. (3) as

$$E_{\rm DGTR} = \frac{\langle 0|[O_+^2, [\mathcal{H}, O_-^2]]|0\rangle}{\langle 0|O_+^2 O_-^2|0\rangle}, \qquad (5)$$

where the GT transition operators are $O_\pm = \sum_i^A \sigma_z(i)t_\pm(i)$. After some algebraic manpulations, we can rewrite the energy of DGTR (5) as

$$E_{\rm DGTR} = 2E_{\rm GT} + \frac{\langle 0|[O_+, [O_+, [[H, O_-], O_-]]]|0\rangle}{2(N-Z)(N-Z-1)} \ . \qquad (6)$$

Double GT and IAS average excitation energies have been determined for the first time using double and quartic commutator relations. In order to provide semi-quantitative theoretical estimates, we have adopted two approximations. First, an independent particle picture has been assumed. We have also provided expressions in which, by simplifying further, the neutron and proton distributions have been taken as hard spheres. This simplification has turned out to be very useful to capture the main terms dominating the calculated quantities.

In conclusion, within our approach, double IAS and GT resonance energies in neutron-rich nuclei are dominated by the same physics as their single counterparts because the main contribution is from $2E_{\rm IAS}$ and $2E_{\rm GTR}$, respectively. Hence, the effect of two-body Coulomb interaction has a decisive effect on the average energy $E_{\rm DIAS}$, while the spin-orbit and residual isospin and spin-isospin interactions play a substantial role in the average energy $E_{\rm DGTR} - E_{\rm DIAS}$. More specifically, we have found that the corrections due to quartic commutators follow the approximate laws $E_{\rm DIAS} - 2E_{\rm IAS} \approx \frac{3}{2}A^{-1/3}$ MeV (even the isospin mixing effects are accounted in $E_{\rm IAS}$) and $E_{\rm DGTR} - E_{\rm DIAS} - 2(E_{\rm GTR} - E_{\rm IAS}) \approx 16A^{-1}$ MeV.

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Reference

1) M. Takaki *et al.*, Proposal for Nuclear Physics Experiment at RI Beam Factory, "Search for double Gamow-Teller giant resonances in $\beta\beta$-decay nuclei via the heavy-ion double charge exchange ^{48}Ca(^{12}C, ^{12}Be(0_2^+)) reaction."