Effects of finite nucleon size, vacuum polarization, and electromagnetic spin-orbit interaction on nuclear binding energies and radii in spherical nuclei†

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In the density functional theory (DFT) for nuclear physics, the ground-state energy is usually given by $E_{\mathrm{gs}} = T_0 + E_{\mathrm{nucl}}[\rho_p, \rho_n] + E_{\mathrm{Cd}}[\rho_{\mathrm{ch}}] + E_{\mathrm{Cx}}[\rho_{\mathrm{ch}}]$, where T_0 is the Kohn-Sham kinetic energy and E_{nucl}, E_{Cd}, and E_{Cx} are the energy density functionals (EDFs) of the nuclear, Coulomb direct, and Coulomb exchange parts, respectively. Here, ρ_p and ρ_n are the ground-state density distributions of protons and neutrons, respectively, and ρ_{ch} is the charge density distribution.

The Coulomb EDF $E_{\mathrm{C}} = E_{\mathrm{Cd}} + E_{\mathrm{Cx}}$ is, in principle, written in terms of the charge density ρ_{ch}[1)] because the Coulomb interaction involves the charge itself, instead of the point protons. Nevertheless, the protons and neutrons are assumed to be point particles, *i.e.*, $\rho_{\mathrm{ch}} \equiv \rho_p$, in most self-consistent nuclear DFT calculations. In this paper, the finite-size effects of nucleons are implemented to the self-consistent steps of the Skyrme Hartree-Fock calculation, where only the electric form factors of nucleons are considered. The Coulomb direct term E_{Cd} holds the conventional form, while the modified Perdew-Burke-Ernzerhof GGA Coulomb exchange functional [2,3)] is used instead of the exact Fock term.

The charge density distribution ρ_{ch} is written in terms of ρ_p, ρ_n, and the electric form factors of protons and neutrons, $\tilde{G}_{\mathrm{E}p}$ and $\tilde{G}_{\mathrm{E}n}$, as $\tilde{\rho}_{\mathrm{ch}}(q) = \tilde{G}_{\mathrm{E}p}(q^2)\,\tilde{\rho}_p(q) + \tilde{G}_{\mathrm{E}n}(q^2)\,\tilde{\rho}_n(q)$, where the quantities with the tilde denote those in the momentum space. Once the finite-size effects are considered, *i.e.*, $\rho_{\mathrm{ch}} \not\equiv \rho_p$, the chain rule of the functional derivative should be applied to derive the effective potential of the nucleon τ, $V_{\mathrm{eff}\tau}(\boldsymbol{r}) = \frac{\delta E[\rho_p,\rho_n]}{\delta \rho_\tau(\boldsymbol{r})}$. Finally, the Coulomb potential for nucleons with the finite-size effects reads $V_{\mathrm{C}\tau}(r) = \{\mathcal{V}_{\mathrm{C}}[\rho_{\mathrm{ch}}] * G_{\mathrm{E}\tau}\}(r)$, where $\mathcal{V}_{\mathrm{C}}[\rho_{\mathrm{ch}}]$ is the conventional form of the Coulomb potential and $*$ denotes the convolution. The Coulomb potential for the neutrons does not vanish within the self-consistent finite-size effects, because $G_{\mathrm{E}n} \not\equiv 0$.

Other possible electromagnetic (EM) contributions, *i.e.*, the vacuum polarization and EM spin-orbit interaction, are also considered. We used effective one-body potential of the vacuum polarization for a charged particle under the Coulomb potential caused by ρ_{ch}, known as the Uehling potential.[4)] The EM spin-orbit interaction, which originates from the interaction between the nucleon spin and $V_{\mathrm{C}\tau}$, is also considered by using the first-order perturbation theory.

The isospin-symmetry breaking (ISB) terms of the nuclear force are also implemented to compare the contributions of the EM interaction with those of the ISB terms of nuclear force. The SAMi functional[5)] is used for the nuclear EDF for most calculations, and the SAMi-ISB functional[6)] is used instead when the ISB originating from E_{nucl} is considered explicitly.

The proton finite-size effect makes the nuclei more bound; for example, the nucleon finite-size effect decreases the binding energy by 8.2 MeV in ^{208}Pb. In contrast, the neutron finite-size effect makes the nuclei less bound, and its contribution is almost one order of magnitude smaller than the proton contribution, for example, by 1.2 MeV in ^{208}Pb, which is non-negligible in heavy nuclei. The contribution of the vacuum polarization to the total energy is also non-negligible, and it makes the nuclei less bound, for example, by 3.7 MeV in ^{208}Pb. The contribution of the electromagnetic spin-orbit interaction to the total energy is approximately 50 keV. Systematically, the contribution of the ISB terms of the nuclear force to the total energy is comparable to that of the proton finite-size effect in heavy nuclei, while the former is more significant than the latter in light nuclei. The neutron finite-size effect and vacuum polarization are also non-negligible. On the other hand, the contribution of the electromagnetic spin-orbit interaction to the total energy depends on the shell structure.

The mass difference between the mirror nuclei ^{48}Ca and ^{48}Ni was also calculated. All the corrections to the Coulomb functional with the SAMi-ISB functional cooperate to reproduce the mirror-nuclei mass difference within 300 keV accuracy, while the error is 1.5 MeV if the conventional Coulomb functional is used.

References

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