Second and fourth moments of the charge density and neutron-skin thickness of atomic nuclei†

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Recently, the fourth moment of the charge distribution has been highlighted as a possible proxy to access information of the neutron root-mean-square radius.[1–3] This is because the neutron distributions ρ_n of atomic nuclei do contribute to their charge density distributions ρ_{ch}. In other words, precise measurements of ρ_{ch} may provide information on ρ_n as well as ρ_p. For instance, Ref. 3) showed the feasibility to extract $\left\langle r^2 \right\rangle_n$ using $\left\langle r^2 \right\rangle_{\mathrm{ch}}$ and $\left\langle r^4 \right\rangle_{\mathrm{ch}}$. This, however, relies on a correlation for specific nuclei based on a specific type of models, and hence, it is questionable whether that method can be applied, in general.

To answer this question, in this paper, we discuss the feasibility of extracting $\left\langle r^2 \right\rangle_n$ from the second and fourth moments of ρ_{ch}, avoiding as much as possible the use of model-induced correlations. We also explore a method to extract the neutron-skin thickness by employing the information of ρ_{ch} of two neighboring even-even isotopes to cancel a large part of the spin-orbit contributions and reduce the uncertainty due to the nucleon form factors and the pairing correlation. We will show that the key issue is to extract $\left\langle r^2 \right\rangle_n$ from $\left\langle r^2 \right\rangle_{\mathrm{ch}}$ and $\left\langle r^4 \right\rangle_{\mathrm{ch}}$ to accurately determine $\left\langle r^4 \right\rangle_p$.

The second and fourth moments of ρ_{ch} read

$$\left\langle r^2 \right\rangle_{\mathrm{ch}} = \left\langle r^2 \right\rangle_p + \left(r^2_{\mathrm{E}p} + \frac{N}{Z} r^2_{\mathrm{E}n} \right) + \left\langle r^2 \right\rangle_{\mathrm{SO}p} + \frac{N}{Z} \left\langle r^2 \right\rangle_{\mathrm{SO}n}, \tag{1a}$$

$$\left\langle r^4 \right\rangle_{\mathrm{ch}} = \left\langle r^4 \right\rangle_p + \frac{10}{3} \left(r^2_{\mathrm{E}p} \left\langle r^2 \right\rangle_p + \frac{N}{Z} r^2_{\mathrm{E}n} \left\langle r^2 \right\rangle_n \right) + \left(r^4_{\mathrm{E}p} + \frac{N}{Z} r^4_{\mathrm{E}n} \right) + \left\langle r^4 \right\rangle_{\mathrm{SO}p} + \frac{N}{Z} \left\langle r^4 \right\rangle_{\mathrm{SO}n}, \tag{1b}$$

where $r^2_{\mathrm{E}\tau}$ and $r^4_{\mathrm{E}\tau}$ are the second and fourth moments of the charge distribution of the nucleon τ, respectively. The spin-orbit contributions $\left\langle r^n \right\rangle_{\mathrm{SO}\tau}$ read

$$\left\langle r^2 \right\rangle_{\mathrm{SO}\tau} \simeq \frac{\kappa_\tau}{M^2_\tau N_\tau} \sum_{a \in \mathrm{occ}} \mathcal{N}_{a\tau} \left\langle \boldsymbol{l} \cdot \boldsymbol{\sigma} \right\rangle, \tag{2a}$$

$$\left\langle r^4 \right\rangle_{\mathrm{SO}\tau} \simeq \frac{10}{M^2_\tau N_\tau} \left[\frac{\kappa_\tau}{5} \left\langle r^2 \right\rangle_\tau + \frac{r^2_{\mathrm{M}\tau} - r^2_{\mathrm{E}\tau}}{3} + \frac{\kappa_\tau}{2M^2_\tau} \right] \times \sum_{a \in \mathrm{occ}} \mathcal{N}_{a\tau} \left\langle \boldsymbol{l} \cdot \boldsymbol{\sigma} \right\rangle, \tag{2b}$$

where $N_\tau = Z$ for the proton ($\tau = p$) distribution and $N_\tau = N$ for the neutron ($\tau = n$) distribution, κ_τ is the anomalous magnetic moment of the nucleon τ, and $r^2_{\mathrm{M}\tau}$ is the second moment of magnetic distribution of the nucleon τ. The index $a = (n, \kappa, m)$ is the set of quantum numbers of a single-particle orbital, the occupation number of which is $\mathcal{N}_{a\tau}$. It is also assumed that the second moment of a single-particle orbital is approximately equal to $\left\langle r^2 \right\rangle_\tau$, which is probably a good approximation except in weakly bound systems.

We then consider whether $\left\langle r^2 \right\rangle_n$ can be extracted from experimental data of $\left\langle r^2 \right\rangle_{\mathrm{ch}}$ and $\left\langle r^4 \right\rangle_{\mathrm{ch}}$ by using Eqs. (1a) and (1b). Our goal was to reduce model assumptions to a minimum. To this end, we have discussed in detail two contributions to the neutron moment: $\left\langle r^n \right\rangle_{\mathrm{SO}\tau}$ and $\left\langle r^4 \right\rangle_p$. As for the latter, we have seen that it can be related to $\left\langle r^2 \right\rangle_p$ in a quite robust manner. Therefore, we deem that we have been able to determine the mildest assumptions under which the $\left\langle r^2 \right\rangle_n$ of a single isotope can be extracted.

Our main result has been the introduction of a novel method to extract $\left\langle r^2 \right\rangle_n$ from ρ_{ch} using the information of two neighboring even-even nuclei. The uncertainties due to the nucleon form factors and the approximation for $\left\langle r^n \right\rangle_{\mathrm{SO}\tau}$ are suppressed, whereas the uncertainties introduced by the pairing are negligible. We advocate that this method, namely the consideration of two neighboring isotopes, is more reliable.

Despite our efforts, we conclude that the main obstacle to an accurate determination of the neutron radius is the contribution from $\left\langle r^4 \right\rangle_p$. Even if it is strongly correlated to $\left\langle r^2 \right\rangle_p$, the resulting uncertainty cannot be neglected. This uncertainty is strongly enhanced when propagated from the fourth moment to the neutron radius. Eventually, extracting the neutron radius or the neutron-skin thickness from $\left\langle r^2 \right\rangle_{\mathrm{ch}}$ and $\left\langle r^4 \right\rangle_{\mathrm{ch}}$ does not seem to be feasible based on the present discussion. Despite these pessimistic conclusions, the equations derived in this paper may be useful for further understanding and investigation and even more useful if, in the future, a clever way to better determine the proton fourth moment can be envisaged.

References
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