# Comparative study on charge radii and their kinks at magic numbers†

T. Naito,[*1,*2] T. Oishi,[*3] H. Sagawa,[*4,*3] and Z. Wang[*5,*6]

The charge radius $R_{\rm ch}$ is one of the important properties of atomic nuclei. The slope of $R_{\rm ch}$ as a function of the neutron number $N$ is known to change at the neutron magic number, which is called the "kink" behavior.[1–3)]

In this paper, we discuss whether the kink behavior can be reproduced even in the "normal" treatment, that is, mean-field models with a widely used functional and standard contact pairing interaction, without any additional terms or effects. We compare $R_{\rm ch}$ of Sn and Pb isotopes obtained by the nonrelativistic Skyrme Hartree-Fock (SHF), relativistic mean-field (RMF), and relativistic Hartree-Fock (RHF) calculations. This comparison enables us to discuss the kink behavior from the perspectives of spin-orbit splitting.

We found that the RHF calculations generally provide a larger kink than that of RMF. This can be understood as follows: the spin-orbit interaction is important for the kink behavior.[2)] The spin-orbit mean field appears from both the relativistic effect and tensor interaction in RHF calculations, while only the former contributes in RMF.

The mass-number, $A$, dependence of the difference between the root-mean-square charge radius of $^{A}$Sn and that of $^{132}$Sn, $R^{\rm Sn}_{\rm ch}(A) - R^{\rm Sn}_{\rm ch}(132)$, calculated in RHF energy density functionals (EDFs) are shown in Fig. 1. Among the RHF calculations, the kink size of $PKO_2$ is the weakest and PKO3 and PKO1* give appreciable kinks. The PKO1, PKO2, PKO3, PKO1*, and PKA1 start from the same Lagrangian, while the number of mesons and their meson-nucleon coupling constants are different. This suggests that $\pi$- and $\rho$-meson exchange, which give the tensor interaction, are important to reproduce the kink behavior. According to Ref. 4), the strength of the tensor interactions of PKO2 is the weakest and that of PKO1* is the strongest; that of PKO3 is the second strongest, and those of PKA1 and PKO1 are marginal. Strengths of the tensor interaction of these RHF models are consistent with the kink size: the stronger the tensor interaction the larger the kink size. Therefore, the tensor interaction or its outcomes, such as spin-orbit mean field and shell structure, are important to reproduce the kink behavior.

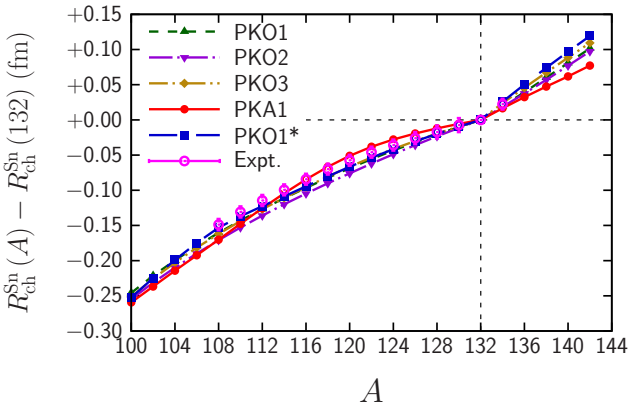


Fig. 1. The mass-number $A$ dependence of $R^{\rm Sn}_{\rm ch}(A) - R^{\rm Sn}_{\rm ch}(132)$, calculated in relativistic Hartree-Fock EDFs. Experimental data (Expt.) are taken from Refs. 6) and 7).

In contrast, the spin-orbit interaction in the Skyrme calculation is introduced phenomenologically as the $W_0$ and $W'_0$ terms without any density dependence. Among the Skyrme interactions tested, SGII and SAMi give reasonable kink behavior. Refs. 4) and 5) claim that the introduction of the isoscalar and isovector spin-orbit interactions, $W_0$ and $W'_0$, is essential, while the standard Skyrme EDFs only have the isoscalar term ($W_0 = W'_0$). However, we found that among better EDFs to describe the kink behavior, the two strengths are identical, $W_0 = W'_0$ in the SGII EDF. In the SkI4 EDF, the isoscalar and isovector spin-orbit interactions have opposite signs, specifically, $W'_0 < 0 < W_0$ and $|W'_0| < W_0$. In the SAMi EDF, the isoscalar and isovector spin-orbit interactions are of the same sign, but the strengths are different, $0 < W'_0 < W_0$. Since the strengths and signs of $W_0$ and $W'_0$ are different, it is not possible at the moment to identify which combination of $W_0$ and $W'_0$ values are the best for the kink behavior.

References
1) P.-G. Reinhard *et al.*, Nucl. Phys. A **584**, 467 (1995).
2) H. Nakada, Phys. Rev. C **100**, 044310 (2019).
3) U. C. Perera *et al.*, Phys. Rev. C **104**, 064313 (2021).
4) Z. Wang *et al.*, Phys. Rev. C **103**, 064326 (2021).
5) M. M. Sharma *et al.*, Phys. Rev. Lett. **74**, 3744 (1995).
6) I. Angeli and K. P. Marinova, At. Data Nucl. Data Tables **99**, 69 (2013).
7) C. Gorges *et al.*, Phys. Rev. Lett. **122**, 192502 (2019).

† Condensed from the article in Phys. Rev. C **107**, 054307 (2023)

*1 RIKEN iTHEMS Program
*2 Department of Physics, University of Tokyo
*3 RIKEN Nishina Center
*4 Center for Mathematics and Physics, University of Aizu
*5 School of Nuclear Science and Technology, Lanzhou University
*6 Frontiers Science Center for Rare Isotopes, Lanzhou University

DOI:10.34448/RIKEN.APR.57-016