

CRDW Manual

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1 What is CRDW

The computer program CRDW calculates the nucleon induced inelastic and charge exchange reactions to continuum as well as discrete states by means of a distorted wave impulse approximation (DWIA). It treats nuclear states by a continuum random phase approximation (CRPA). CRDW is the abbreviation for CRPA + DWIA.

The code is written in FORTRAN. This program is originated from the program coded by Izumoto in 1980. Since then, it has been extended to various directions by Ken Kawahigashi, Thomas Sams, Kimiaki Nishida, Yasushi Nakaoka, Tomotugu Wakasa and Munetake Ichimura[1]-[7]. Ichimura takes full responsibility throughout the development.

2 What can CRDW calculate

2.1 Reactions

CRDW calculates the nuclear reactions

$$A(N, N')X \tag{2.1}$$

where

N : Projectile nucleon (proton (p) or neutron (n))

N' : Ejectile nucleon (p or n)

A : Target nucleus, which is restricted to the doubly closed shell nuclei.

X : Residual nuclear system which can include the isobar Δ .

Typical examples are

- | | |
|---|--|
| (1) Quasi-elastic scattering (QES) | eg. $^{12}\text{C}(p, n)$ |
| (2) Continuum state excitation with the fixed J^π | eg. $^{90}\text{Zr}(p, n)(1^+)$ |
| (3) Discrete state excitation with the fixed J^π | eg. $^{16}\text{O}(p, p')^{16}\text{O}(0^-)$ |

2.2 Calculated quantities

Details of output data are explained in sect.7. Main outputs are the followings.

2.2.1 Reaction observables

- | | |
|--|--|
| (1) Unpolarized double-differential cross sections | $I(\theta, \omega) = \frac{d^2\sigma}{d\omega d\Omega}$ |
| (2) Polarization and analyzing power | $P(\theta, \omega), A_y(\theta, \omega)$ |
| (3) Polarization transfer coefficients | $D_{ij}(\theta, \omega)$ |
| (4) Spin-polarized cross sections | $ID_0(\theta, \omega), ID_n(\theta, \omega)$
$ID_q(\theta, \omega), ID_p(\theta, \omega)$ |

where θ and ω are the scattering angle and the energy transfer, respectively.

2.2.2 Response functions

- | | |
|---|-------------------|
| (1) Spin-scalar response function | $R_S(q', \omega)$ |
| (2) Spin-longitudinal response function | $R_L(q', \omega)$ |
| (3) Spin-transverse response function | $R_T(q', \omega)$ |
| (4) Spin-vector response function | $R_V(q', \omega)$ |

where q' and ω are the intrinsic momentum and energy transfers.

2.3 Theories

2.3.1 Reaction mechanism

CRDW adopts DWIA as the reaction mechanism. The present version can calculate only isovector transitions, namely, those induced by the $\boldsymbol{\tau} \cdot \boldsymbol{\tau}$ -parts of the NN t-matrix.

2.3.2 Structure theory

CRDW treats the nuclear structure by the following theoretical frameworks.

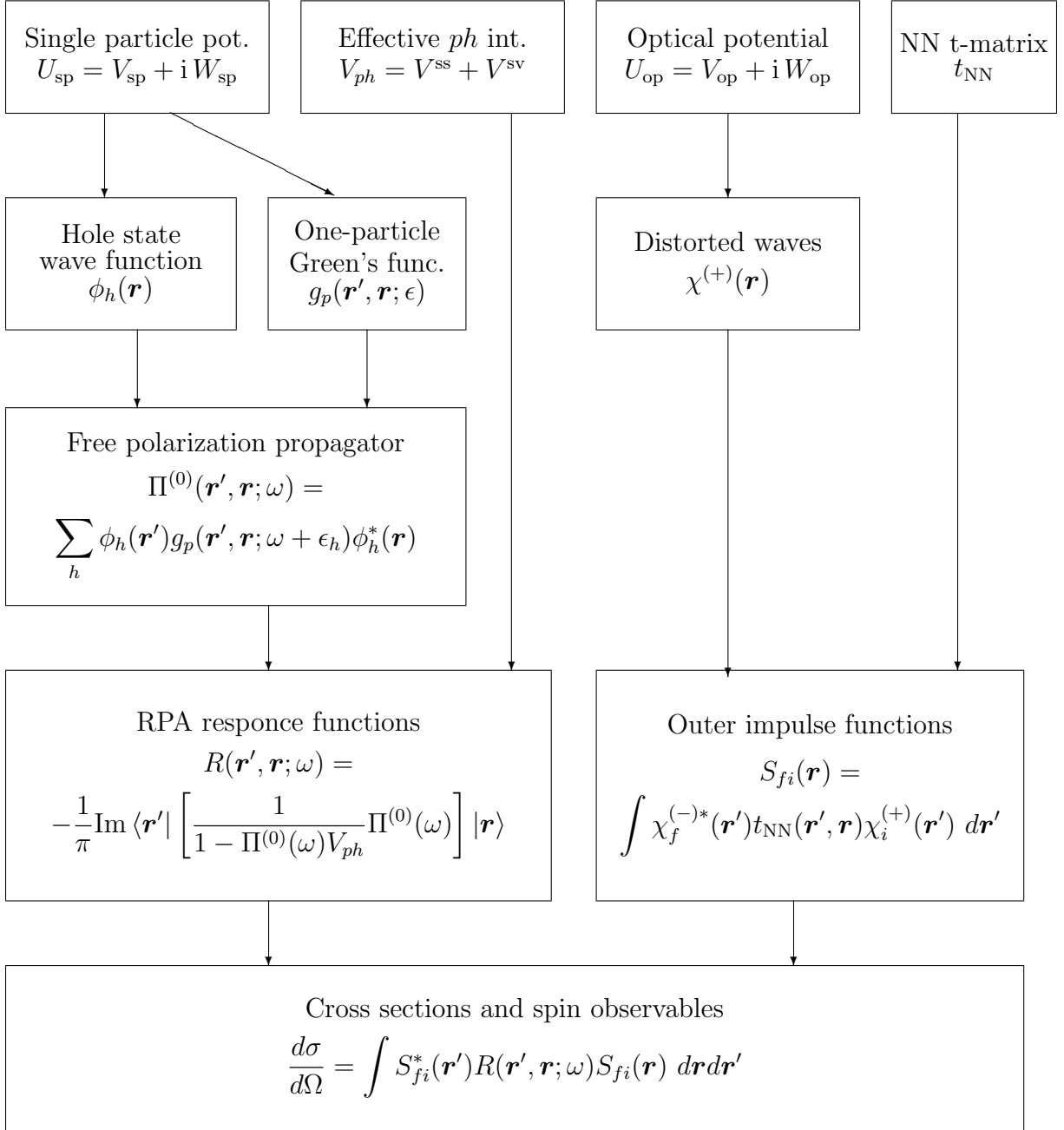
- (1) A single particle model in a (complex) Woods-Saxon potential with effective masses and spreading widths
- (2) A continuum RPA and TDA with the orthogonality condition for nuclear correlations in the isovector channels
- (3) The f' Landau-Migdal interaction for the isovector spin-scalar channel
- (4) The $\pi + \rho + g' + h'$ interaction for the isovector spin-vector channel
- (5) The Δ degree of freedom can be included.

2.3.3 Formalism

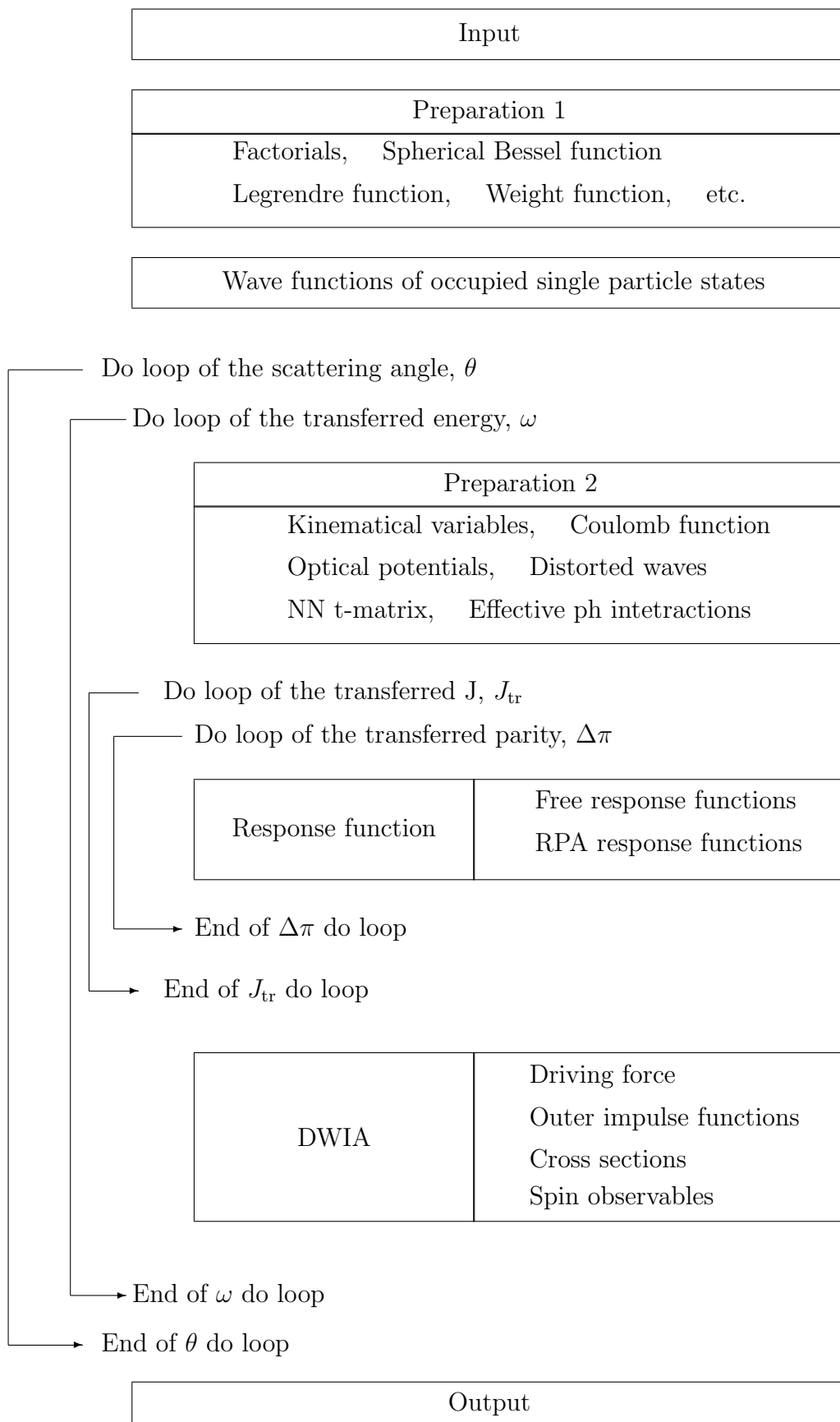
Formalism is presented in the document 'Formalism of CRDW'.

3 Structure of the program

3.1 Outline of calculation



3.2 Flow of the program



Details of the program structure with subprogram names are given in Appendix A

3.3 Compilation and run

An example of the shellsript to compile and run CRDW by g77 on c-shell of Linux

A. Compile

Execute the command "make" to work the following Makefile in the directory /src.

```
# Makefile for RPA+DWIA programs
#
FC          = g77
FFLAGS     = -O2 -fno-automatic -finit-local-zero
INCLUDE    = ../include
RM         = rm
OBJS       = acal.o ampcla.o ampsub.o basic3.o bfcad.o \
             bound.o c1cal.o c2cal.o cleb.o clnqgj.o \
             coulfn.o couplesv.o couples.o couple2.o cxbd.o cxfh.o \
             diswav.o drcopt.o ecal.o fcal.o fnsg2.o glopt2.o \
             gqq.o input0.o inte3.o alegdm.o lgauss.o lgdr.o \
             lgsbsl.o main.o opfact.o nnamp.o nnamp0.o nonrpa.o nrelopt.o \
             ocmg.o overlap.o pgen2.o rac.o rsrtrl.o sbessl.o sigma.o \
             slfzr.o sobcal.o sp0cp.o spcp.o spwd.o stfcfg.o phspec.o \
             u9.o vdrll.o vdrll_fl.o veffsv.o \
             wbound.o wconv.o weight.o wNNamp.o write0.o wresp.o \
             wxpol.o wxsecJ.o xpol.o \
             coulp3.o edad1.o edaic.o edaipb.o fcj.o global.o \
             deri6.o edad2.o edaica.o edaizr.o osucou.o \
             dgi16.o edad3.o edaio.o fci.o glob12.o glopt3.o hepb1.o \
             weffq.o globhe.o nnamp_fl.o fxpot.o BplusFcal.o

.f.o:
    ${FC} ${FFLAGS} -I ${INCLUDE} -c $ <
main: ${OBJS}
    ${FC} ${FFLAGS} -o $@ ${OBJS}
all: main
clean:
    ${RM} main *.o
```

B. Run

Link the unit 1 to the input data file.

Link the unit 10 to the NN-t-matrix data file when the optimal factorization is used.

Then run!

```
----- Shellsript example -----
# ! /bin/csh
ln -s input/nnamp/nnamp92.dat fort.10
ln -s 'input data file name' fort.1
./main > 'output file name'
\rm fort.1
\rm fort.10
```

4 How to make input data

4.1 Outline of input form

4.2 Structure

line 1	Date, Run-number
line 2	Title
line 3	Calculation options
line 4	Output options
line 5 and below	Page style input : Data with page and subpage
last line	Blank: end of input data

4.3 Contents of data with page and subpage

page	subpage	content
1	-	Mass number, Charges
2	-	Incident energy, Transferred energies, Scattering angles
3	-	Transferred angular momenta and transferred parity
4	-	Data for integration
5	1-5	Single particle potential parameters (real part)
6	1-5	Single particle potential parameters (imaginary part)
7	1-4	Effective mass and spreading width parameters
8	1, 2	Number of single particle states, followed by table of state specification (<i>nlj</i>)
9	0-5	Effective <i>ph</i> interaction parameters
10	-	Partial wave data, Optical potential specification
11	1,2	Non-relativistic optical potential parameters (real part)
12	1,2	Non-relativistic optical potential parameters (imaginary part)
13	1,2	Non-relativistic optical potential parameters (surface part)
14	1,2	Dirac type optical potential parameters (real part)
15	1,2	Dirac type optical potential parameters (imaginary part)

page.11-16 are not used if Global optical potential is selected.

4.4 Detaild description of input data

4.4.1 Date, run number and title

Input format

```
read(1,'(2I2, I4, I2)') (numrun(i),i=1,4)
read(1,'(20a4)') ititle
```

numrun(1)	month
numrun(2)	day
numrun(3)	year
numrun(4)	run number
ititle	Title within 100 characters

4.4.2 Calculation options

Input format

```
read(1,'(10I1)') KTCAL
```

Variable Name	Input Variable	Value	Explanation
KTDW	KTCAL(1)	0	PWIA
		1	DWIA
		2	Response function only
KTRPA	KTCAL(2)	0	0th
		1	TDA
		2	RPA
		3	RPA for p-h but TDA for Δ -h
KTND	KTCAL(3)	0	without Δ
		1	with Δ
KTRQCV	KTCAL(4)	0	Calculate $R(q, \omega)$. No convergence check
		1	Calculate $R(q, \omega)$. Convergence check for R_S and R_V
		2	Calculate $R(q, \omega)$. Convergence check for R_S
		3	Calculate $R(q, \omega)$. Convergence check for R_V
		4	Not calculate $R(q, \omega)$
KTTNN	KTCAL(5)	0	Optimal factorization [8]
		1	Franey-Love t-matrix [9]
KTAMP	KTCAL(6)		Specify the KMT amplitudes included ^{*)}
		0	All amplitudes
		1	A
		2	C1
		3	C2
		4	E
		5	F
		6	B-F
		7	A+E+(B+F)/2 (central + tensor)
		8	A+C1 ($S_{tr} = 0$ excitation dominant)
9	E+F+(B-F)+C2 ($S_{tr} = 1$ excitation dominant)		
KTVEFF	KTCAL(7)	0	Only LM term for effective ph interaction
		1	LM + OBEP for the effective ph interaction
KTHSPWD	KTCAL(8)	0	without spreading widths of holes
		1	with spreading widths of holes

^{*)} Note ! The option KTAMP = 0, 2, 3, 6, 8 and 9 utilize the fixed reaction plane approximation, which does not work at $\theta = 0$ because the reaction plane cannot be defined. Avoid these options at very small angles, but use the option KTAMP=7.

4.4.3 Output options

Input Form

```
read(1,'(10I1)') KTOUT
```

Variable Name	Input Variable	Value	Explanation
KTOUT_RQ	KTOUT(1)	0	No output of $R(q, \omega)$
		1	Output $R(q, \omega)$
		2	Output weighted $R^{N/\Delta, N/\Delta}(q, \omega)$ if KTND=1
KTOUT_VEF	KTOUT(2)	0	No output of the effective ph interactions
		1	Output the effective ph interactions
KTOUT_BDW	KTOUT(3)	0	No output of the bound state wave function
		N	Output the bound state wave func. in step of N
KTOUT_XS	KTOUT(4)	0	Standard output of $\sigma, P, A, D_{ij}, ID_i$
		1	Contributions from NN, N Δ , $\Delta\Delta$, separately
KTOUT_XSJ	KTOUT(5)	0	No output of $\sigma(J^\pi)$
		1	Output $\sigma(J^\pi)$
KTOUT_OPT	KTOUT(6)	0	No output of the optical potentials
		1	Output the optical potentials
KTOUT_DW	KTOUT(7)	0	No output of the distorted waves
		1	Output the distorted waves
KTOUT_NNA	KTOUT(8)	0	No output of the NN t-matrix amplitudes
		1	Output the squares of the NN t-matrix amplitudes
KTOUT_LTRATIO	KTOUT(9)	0	No output of the L/T ratio
		1	Output of the L/T ratio
KTOUT_PROP	KTOUT(10)	0	No output of IDi/Ri ratio
		1	Output of IDi/Ri ratio if KTOUT_RQ=1

4.4.4 Page style input

Input format

```
read(1,'(8f10.5)') (a(i),i=1,8)
page I=a(1), subpage ICH =(a(1)-I)*10
```

Explanations below are given in the form of

- 1st column : Internal variable name
- 2nd column : Input variable name
- 3rd column : Explanation
- 4th column : Notation in the document of the formalism

PAGE 1 Mass and charges

1.0	a(1)	Page	
Atg	a(2)	Target mass number	A
Ztg	a(3)	Target charge	Z_A
Zprj	a(4)	Projectile charge	Z_N
Zejc	a(5)	Ejectile charge	$Z_{N'}$

PAGE 2 Incident energy, scattering angles and transferred energies

2.0	a(1)	Page	
elabi	a(2)	Incident kinetic energy in lab. (MeV)	K_{lab}
nanmax	a(3)	Number of scattering angles. (\leq nan dm)	N_{θ}^{max}
thetlmin	a(4)	Minimum scattering angle in lab. (deg)	$\theta_{\text{lab}}^{\text{min}}$
dthetl	a(5)	Increment of scattering angle in lab. (deg)	$\Delta\theta_{\text{lab}}$
nwmax	a(6)	Number of transferred energies. (\leq nw dm)	N_{ω}^{max}
omglmin	a(7)	Minimum transferred energy in lab. (MeV)	$\omega_{\text{lab}}^{\text{min}}$
domegl	a(8)	Increment of transferred energy in lab. (MeV)	$\Delta\omega_{\text{lab}}$

*) nw dm is defined in include/dimm.fh/ nw dm must be less than or equal to 51.

PAGE 3 Transferred angular momenta J_{tr} , transferred parities $\Delta\pi$, and energy shift

3.0	a(1)	Page	
itrjmn	a(2)	Minimum transferred total angular momentum	$J_{\text{tr}}^{\text{min}}$
itrjmx	a(3)	Maximum transferred total angular momentum	$J_{\text{tr}}^{\text{max}}$
knmin	a(4)	Minimum parity transfer index kn	
knmax	a(5)	Maximum parity transfer index kn	
KTLTR	a(6)	Orbital angular momentum transfer option = 0: Standard = 1: Only lowest L_{tr} for the given J_{tr} and $\Delta\pi$	
EPSJ	a(7)	Criterion for the convergence check of the response functions with respect to J_{tr} . If a(7)=0, EPSJ=0.005 (default)	
WSHIFT	a(8)	Artificial energy shift for response function (MeV)	ω_{shift}

*) kn=1: natural parity excitation, kn=2: unnatural parity excitation

PAGE 4 Integration parameters

4.0	a(1)	Page	
rmax	a(2)	Maximum radius, r (fm)	r_{max}
nsrmax	a(3)	Number of crude mesh* points. (\leq nsr dm)	N_{sr}^{max}
tkdmx	a(4)	Maximum momentum, k for Fourier transformation (fm^{-1})	k_{max}
nkmax	a(5)	Number of mesh points for k . (\leq nk dm)	N_k^{max}

*) Fine mesh dr , crude mesh $d_{sr} = r_{\text{max}}/N_{sr}^{\text{max}} = 2dr$

*) nsr dm and nk dm are defined in include/dimm.fh/

PAGE 5 Single particle potential (real part)

5.ich	a(1)	Page.subpage ich=1: neutron hole, =2: proton hole, =3: neutron particle, =4: proton particle, =5: Δ	
vd(ich)	a(2)	Central potential depth (MeV). If vd=0, it is calculated from the given binding energy of the top state.	V_0^{α}
rrd(ich)	a(3)	Central potential radius parameter (fm)	r_{vr}^{α}
ard(ich)	a(4)	Central potential diffuseness parameter (fm)	a_{vr}^{α}
vsod(ich)	a(5)	Spin-orbit potential strength (MeV)	V_{so}^{α}
rsod(ich)	a(6)	Spin-orbit potential radius parameter (fm)	r_{so}^{α}
asod(ich)	a(7)	Spin-orbit potential diffuseness parameter (fm)	a_{so}^{α}
rcd(ich)	a(8)	Coulomb radius parameter (fm)	r_c^{α}

*) $\alpha = \text{ich} = n^{-1}, p^{-1}, n, p$, or Δ

PAGE 6 Single particle potential (imaginary part)

6.ich	a(1)	Page.subpage ich =3: neutron particle, =4: proton particle, =5: Δ	
wd(ich)	a(2)	Central potential depth (MeV). If wd=999.0, calculated by the spreading width formula. If wd is less than 0.1MeV, it is set to be 0.1MeV.	W_0^α
rid(ich)	a(3)	Central potential radius parameter (fm)	r_{vi}^α
aid(ich)	a(4)	Central potential diffuseness parameter (fm)	a_{vi}^α
WV0(ich)	a(5)	Relative strength of the volume type	W_v^α
WS0(ich)	a(6)	Relative strength of the surface type	W_s^α
RVD2(ICH)	a(7)	Surface potential radius parameter (fm)	r_{si}^α
AVD2(ICH)	a(8)	Surface potential diffuseness parameter (fm)	a_{si}^α

*) $\alpha = \text{ich} = n, p, \text{ or } \Delta$

PAGE 7 Perey factor and spreading width parameters

7.ich	a(1)	Page.subpage ich=1: neutron hole, =2: proton hole, =3: neutron particle, =4: proton particle	
reffd(ich)	a(2)	Perey factor radius parameter (fm)	r_{eff}^α
aeffd(ich)	a(3)	Perey factor diffuseness parameter (fm)	a_{eff}^α
bdifd(ich)	a(4)	Strength parameter of the Perey factor	b^α
alphsw(ich)	a(5)	Spreading width strength	$\alpha_{\text{spw}}^\alpha$
eps0sw(ich)	a(6)	Spreading width cutoff parameter 1 (MeV)	ε_0^α
eps1sw(ich)	a(7)	Spreading width cutoff parameter 2 (MeV)	ε_1^α
BEFM(ich)	a(8)	Absolute value of Fermi energy	$ \epsilon_F^\alpha $

*) $\alpha = \text{ich} = n^{-1}, p^{-1}, n \text{ or } p$

PAGE 8 Occupied single particle states

8.ich	a(1)	Page.subpage ich=1: neutron hole, =2: proton hole
nspm(ich)	a(2)	Number of occupied single particle states

Followed by reading the single particle state information with input format
do n=1, nspm(ich)
read(1,'(2I5,2f10.5)') nspd(n,ich), lspd(n,ich),
1 FJSPD(N,ICH), BERSPD(N,ICH)
end do

nspd(n,ich)	Nodal quantum number	n^α
lspd(n,ich)	Orbital angular momentum	l^α
FJSPD(N,ICH)	Total angular momentum	j^α
BERSPD(N,ICH)	Binding energy (MeV)	E_r^α

*) $\alpha = \text{ich} = n^{-1} \text{ or } p^{-1}$

9.0	a(1)	Spin-scalar contact interaction	
Vtautau	a(2)	Isovector spin-scalar interaction parameter (MeV fm ³)	V_τ

9.1	a(1)	π, ρ parameters	
facbs	a(2)	π NN coupling constant	$f_{\pi NN}^2/(4\pi)$
fratio	a(3)	π N Δ , π NN coupling constant ratio	$f_{\pi N\Delta}/f_{\pi NN}$
crhoo	a(4)	ρ, π coupling ratio	C_ρ

9.2	a(1)	One-pion exchange interaction parameters	
epion	a(2)	Pion mass (MeV)	m_π
	a(3)	not used	
ctpi(1)	a(4)	Cut-off mass for NN interaction (MeV)	$\Lambda_{\pi NN}$
nctpi(1)	a(5)	Power of form factor for NN interaction	$n_{\pi NN}$
ctpi(2)	a(6)	Cut-off mass for N Δ interaction (MeV)	$\Lambda_{\pi N\Delta}$
nctpi(2)	a(7)	Power of form factor for N Δ interaction	$n_{\pi N\Delta}$

9.3	a(1)	One- ρ exchange interaction parameters	
erhoo	a(2)	ρ -meson mass (MeV)	m_ρ
	a(3)	not used	
ctrh(1)	a(4)	Cut-off mass for NN interaction (MeV)	$\Lambda_{\rho NN}$
nctrh(1)	a(5)	Power of form factor for NN interaction	$n_{\rho NN}$
ctrh(2)	a(6)	Cut-off mass for N Δ interaction (MeV)	$\Lambda_{\rho N\Delta}$
nctrh(2)	a(7)	Power of form factor for N Δ interaction	$n_{\rho N\Delta}$

9.4	a(1)	g' interaction parameters	
gpr(1,1)	a(2)	g'_{NN}	g'_{NN}
gpr(1,2)	a(3)	$g'_{N\Delta}$	$g'_{N\Delta}$
gpr(2,2)	a(4)	$g'_{\Delta\Delta}$	$g'_{\Delta\Delta}$
ctgpr	a(5)	Cutoff mass for g' form factor (MeV)	Λ_g
nctgpr	a(6)	Power of g' form factor	n_g

9.5	a(1)	h' interaction parameters	
hpr(1,1)	a(2)	h'_{NN}	h'_{NN}
hpr(1,2)	a(3)	$h'_{N\Delta}$	$h'_{N\Delta}$
hpr(2,2)	a(4)	$h'_{\Delta\Delta}$	$h'_{\Delta\Delta}$
cthpr	a(5)	Cutoff mass for h' form factor (MeV)	Λ_h
ncthpr	a(6)	Power of h' form factor	n_h

10.0	a(1)	Partial wave, optical potential and driving force	
lparmn	a(2)	Minimum partial wave orbital angular momentum	l_{\min}
lparmx	a(3)	Maximum partial wave orbital angular momentum	l_{\max}
ktdrc(1)	a(4)	Optical potential type of incident channel	
ktglb(1)	a(5)	Parameter set index of global potential by Cooper et al.[10] for incident channel	
ktdrc(2)	a(6)	Optical potential type of exit channel	
ktglb(2)	a(7)	Parameter set index of global potential by Cooper et al.[10] for exit channel	
EFL	a(8)	Energy used for Franey-Love NN t-matrix [9] If EFL=0, the incident energy is used	

Explanation of ktdrc

ktdrc	Meaning
0	Nonrelativistic optical potential. Parameters are read from page 11-14
1	Dirac phenomenological potential. Parameters are read from page 15, 16
2	Potentials are read from file 15
3	Global relativistic optical potential for proton; Cooper et al. [10]
4	Global relativistic optical potential for neutron; Shen et al. [11]

Explanation of ktglb : specification of the global potential by Cooper et al. [10]

ktglb	Meaning
1	CA40-PB208 (P,P) 65-1040 MEV FIT.1
2	CA40-PB208 (P,P) 65-1040 MEV FIT.2
3	C12 (P,P) 29-1040 MEV (EDAI C12)
4	O16 (P,P) 23-1040 MEV (EDAI O16)
5	CA40 (P,P) 21-1040 MEV (EDAI CA40)
6	ZR90 (P,P) 22-800 MEV (EDAI ZR90)
7	PB208 (P,P) 21-1040 MEV (EDAI PB208)
8	C12-PB208 (P,P) 21-1040 MEV (EDAD FIT.1)
9	C12-PB208 (P,P) 21-1040 MEV (EDAD FIT.2)
10	C12-PB208 (P,P) 21-1040 MEV (EDAD FIT.3)
11	4HE-PB208 (P,P) 21-1040 MEV (UNDEMOCRATIC, TC)
12	4HE-PB208 (P,P) 21-1040 MEV (DEMOCRATIC, SH)
13	4HE (P,P) 156-1728 MEV (PRC73,024608(2006))

PAGE 11 Non-relativistic optical potential parameters (real part)

11.ich	a(1)	Page.subpage: ich=1: incident channel, =2: exit channel	
vd1(ich)	a(2)	Depth of volume type central potential (MeV)	V^N
rrd1(ich)	a(3)	Radius parameter of volume type central potential (fm)	r_r^N
ard1(ich)	a(4)	Diffuseness parameter of volume type central potential (fm)	a_r^N
vsod1(ich)	a(5)	Strength of spin-orbit potential (MeV)	V_{so}^N
rsord(ich)	a(6)	Radius parameter of spin-orbit potential (fm)	r_{sor}^N
asord(ich)	a(7)	Diffuseness parameter of spin-orbit potential (fm)	a_{sor}^N
rcd1(ich)	a(8)	Coulomb radius parameter (fm)	r_c^N

PAGE 12 Non-relativistic optical potential parameters (imaginary part: Volume type)

12.ich	a(1)	Page.subpage: ich=1: incident channel, =2: exit channel	
wd1(ich)	a(2)	Depth of central potential (MeV)	W^N
rid1(ich)	a(4)	Radius parameter of Wood-Saxon potential (fm)	r_{ci}^N
aid1(ich)	a(5)	Diffuseness parameter of Woods-Saxon potential(fm)	a_{ci}^N
wsod(ich)	a(3)	Strength of spin-orbit potential (MeV)	W_{so}^N
rsoid(ich)	a(6)	Radius parameter of spin-orbit potential (fm)	r_{soi}^N
asoid(ich)	a(7)	Diffuseness parameter of spin-orbit potential (fm)	a_{soi}^N

PAGE 13 Non-relativistic optical potential parameters (imaginary part: Surface type)

13.ich	a(1)	Page.subpage: ich=1: incident channel, =2: exit channel	
csdgd(ich)	a(2)	Relative strength of surface type potential	C_{sg}^N
rgd(ich)	a(3)	Radius parameter of Gaussian type potential (fm)	r_g^N
agd(ich)	a(4)	Diffuseness parameter of Gaussian type potential (fm)	a_g^N

PAGE 14 Dirac type optical potential parameters (real part)

14.ich	a(1)	Page.subpage: ich=1: incident channel, =2: exit channel	
vsd(ich)	a(2)	Strength of the scalar potential (MeV)	V_S^N
vvd(ich)	a(3)	Strength of the vector potential (MeV)	V_V^N
rsd(ich)	a(4)	Radius parameter of the scalar potential (fm)	r_{Sr}^N
rvd(ich)	a(5)	Radius parameter of the vector potential (fm)	r_{Vr}^N
asd(ich)	a(6)	Diffuseness parameter of the scalar potential (fm)	a_{Sr}^N
avd(ich)	a(7)	Diffuseness parameter of the vector potential (fm)	a_{Vr}^N
rcd1(ich)	a(8)	Coulomb radius parameter (fm)	r_c^N

PAGE 15 Dirac type optical potential parameters (imaginary part)

15.ich	a(1)	Page.subpage : ich=1: incident channel, =2: exit channel	
wsd(ich)	a(2)	Strength of the scalar potential (MeV)	W_{Si}^N
wvd(ich)	a(3)	Strength of the vector potential (MeV)	W_{Vi}^N
rsid(ich)	a(4)	Radius parameter of the scalar potential (fm)	r_{Si}^N
rvid(ich)	a(5)	Radius parameter of the vector potential (fm)	r_{Vi}^N
asid(ich)	a(6)	Diffuseness parameter of the scalar potential (fm)	a_{Si}^N
avid(ich)	a(7)	Diffuseness parameter of the vector potential (fm)	a_{Vi}^N

*) In this page $N = p, n$.

5 Definition of input parameters

5.1 Single particle potential

The single particle local potentials are written as

$$U^\alpha(\mathbf{r}) = -V_0^\alpha F_{\text{cr}}^\alpha(r) - iW_0^\alpha F_{\text{ci}}^\alpha(r) - V_{\text{so}}^\alpha F_{\text{so}}^\alpha(r) \mathbf{l} \cdot \boldsymbol{\sigma}^\alpha + V_{\text{coul}}^\alpha(r) \quad (5.1)$$

where $\boldsymbol{\sigma}$ is the nucleon Pauli spin operator, and

$$\alpha = \text{ich} = \begin{cases} 1 & n \in \text{occupied states} \\ 2 & p \in \text{occupied states} \\ 3 & n \in \text{unoccupied states} \\ 4 & p \in \text{unoccupied states} \\ 5 & \Delta \end{cases} \quad (5.2)$$

The radial form factors $F_x^\alpha(r)$ and the Coulomb potential $V_{\text{coul}}^\alpha(r)$ are defined as

$$F_{\text{cr}}^\alpha(r) = \frac{1}{1 + \exp[(r - r_{\text{vr}}^\alpha A_c^{1/3})/a_{\text{vr}}^\alpha]} \quad (5.3)$$

$$F_{\text{ci}}^\alpha(r) = \frac{W_{\text{v}}^\alpha}{W_{\text{v}}^\alpha + W_{\text{s}}^\alpha} \frac{1}{1 + \exp[(r - r_{\text{vi}}^\alpha A_c^{1/3})/a_{\text{vi}}^\alpha]} + \frac{W_{\text{s}}^\alpha}{W_{\text{v}}^\alpha + W_{\text{s}}^\alpha} \frac{4 \exp[(r - r_{\text{si}}^\alpha A_c^{1/3})/a_{\text{si}}^\alpha]}{\left(1 + \exp[(r - r_{\text{si}}^\alpha A_c^{1/3})/a_{\text{si}}^\alpha]\right)^2} \quad (5.4)$$

$$F_{\text{so}}^\alpha(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{a_{\text{so}}^\alpha} \frac{1}{r} \frac{\exp[(r - r_{\text{so}}^\alpha A_c^{1/3})/a_{\text{so}}^\alpha]}{\left(1 + \exp[(r - r_{\text{so}}^\alpha A_c^{1/3})/a_{\text{so}}^\alpha]\right)^2} \quad (5.5)$$

$$V_{\text{coul}}^\alpha(r) = \begin{cases} Z_\alpha Z_c \frac{e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2}\right), & (r < R_c = r_c^\alpha A_c^{1/3}) \\ Z_\alpha Z_c \frac{e^2}{r}, & (r \geq R_c) \end{cases} \quad (5.6)$$

where $A_c (= A - 1)$ is the mass number of the core. The present version fixes

$$\left(\frac{\hbar}{m_\pi c}\right)^2 = 2.0 \text{ [fm}^2\text{]} \quad (5.7)$$

5.2 Perey factor

The Perey factors are expressed as

$$P^\alpha(r) = 1 - \frac{b^\alpha}{1 + \exp[(r - r_{\text{eff}}^\alpha A_c^{1/3})/a_{\text{eff}}^\alpha]} \quad (5.8)$$

where $\alpha = \text{ich} = 1-4$. For Δ we set $P^\Delta(r) = 1$.

5.3 Spreading width

The spreading widths for the nucleons are expressed by the phenomenological formula[14]

$$\frac{\gamma^\alpha(\varepsilon)}{2} = \alpha_{\text{spw}}^\alpha \left[\frac{(\varepsilon^\alpha)^2}{(\varepsilon^\alpha)^2 + (\varepsilon_0^\alpha)^2} \right] \left[\frac{(\varepsilon_1^\alpha)^2}{(\varepsilon^\alpha)^2 + (\varepsilon_1^\alpha)^2} \right] \quad (5.9)$$

with

$$\varepsilon^\alpha = \epsilon - \epsilon_F^\alpha \quad (5.10)$$

where $\alpha = \text{ich} = 1-4$, and ϵ_F^α is the Fermi energy

$$\epsilon_F^\alpha = \frac{1}{2} (\epsilon^\alpha \text{ of the highest occupied level} + \epsilon^\alpha \text{ of the lowest unoccupied level}) \quad (5.11)$$

We do not use γ for Δ .

The program has an option to determine the strength of the imaginary potential by

$$W_0^\alpha(\epsilon) = \frac{\gamma(\varepsilon)}{2} \quad (5.12)$$

It also has the option to add the imaginary part to the bound state energy as

$$\epsilon_h^\alpha \longrightarrow \tilde{\epsilon}_h^\alpha = \epsilon_h^\alpha - i \frac{\gamma(\varepsilon_h^\alpha)}{2} \quad (5.13)$$

5.4 Effective ph interactions

The effective ph interaction consists of the isovector spin-scalar interaction V^s and the isovector spin-vector interaction V^v

$$V^{ph} = V^s + V^v \quad (5.14)$$

5.4.1 Isovector spin-scalar interaction

CRDW uses a contact interaction for the isovector spin-scalar modes

$$V_{12}^s(\mathbf{r}_1 - \mathbf{r}_2) = V_\tau (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (5.15)$$

5.4.2 Isovector spin-vector interaction

For the isovector spin-vector modes, the interaction is expressed by the sum of the spin-longitudinal interaction, V^L , and the spin-transverse interaction, V^T , as

$$V_{12}^v = V_{12}^L + V_{12}^T \quad (5.16)$$

They are given in the momentum representation as

$$\begin{aligned} V_{12}^L(\mathbf{q}, \omega) &= W_L^{\text{NN}}(q, \omega) (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) (\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{q}}) (\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{q}}) \\ &+ W_L^{\text{N}\Delta}(q, \omega) [\{(\boldsymbol{\tau}_1 \cdot \mathbf{T}_2) (\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{q}}) (\mathbf{S}_2 \cdot \hat{\mathbf{q}}) + (1 \leftrightarrow 2)\} + \text{h.c.}] \\ &+ W_L^{\Delta\Delta}(q, \omega) \left[\left\{ (\mathbf{T}_1 \cdot \mathbf{T}_2) (\mathbf{S}_1 \cdot \hat{\mathbf{q}}) (\mathbf{S}_2 \cdot \hat{\mathbf{q}}) + (\mathbf{T}_1 \cdot \mathbf{T}_2^\dagger) (\mathbf{S}_1 \cdot \hat{\mathbf{q}}) (\mathbf{S}_2^\dagger \cdot \hat{\mathbf{q}}) \right\} + \text{h.c.} \right] \end{aligned} \quad (5.17a)$$

$$\begin{aligned} V_{12}^T(\mathbf{q}, \omega) &= W_T^{\text{NN}}(q, \omega) (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) (\boldsymbol{\sigma}_1 \times \hat{\mathbf{q}}) \cdot (\boldsymbol{\sigma}_2 \times \hat{\mathbf{q}}) \\ &+ W_T^{\text{N}\Delta}(q, \omega) [\{(\boldsymbol{\tau}_1 \cdot \mathbf{T}_2) (\boldsymbol{\sigma}_1 \times \hat{\mathbf{q}}) \cdot (\mathbf{S}_2 \times \hat{\mathbf{q}}) + (1 \leftrightarrow 2)\} + \text{h.c.}] \\ &+ W_T^{\Delta\Delta}(q, \omega) \left[\left\{ (\mathbf{T}_1 \cdot \mathbf{T}_2) (\mathbf{S}_1 \times \hat{\mathbf{q}}) \cdot (\mathbf{S}_2 \times \hat{\mathbf{q}}) + (\mathbf{T}_1 \cdot \mathbf{T}_2^\dagger) (\mathbf{S}_1 \times \hat{\mathbf{q}}) \cdot (\mathbf{S}_2^\dagger \times \hat{\mathbf{q}}) \right\} \right. \\ &\quad \left. + \text{h.c.} \right] \end{aligned} \quad (5.17b)$$

where \mathbf{S} and \mathbf{T} are the spin and isospin transition operators from N to Δ , respectively.

5.4.3 $\pi + \rho + g' + h'$ model

The program adopts the $\pi + \rho + g' + h'$ model, in which

$$W_L^{NN}(q, \omega) = \frac{f_{\pi NN}^2}{m_\pi^2} \left(g'_{NN} \Gamma_g(t) + 2h'_{NN} \Gamma_h(t) + \frac{q^2}{t - m_\pi^2} \Gamma_{\pi NN}^2(t) \right) \quad (5.18a)$$

$$W_L^{N\Delta}(q, \omega) = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \left(g'_{N\Delta} \Gamma_g(t) + 2h'_{N\Delta} \Gamma_h(t) + \frac{q^2}{t - m_\pi^2} \Gamma_{\pi NN}(t) \Gamma_{\pi N\Delta}(t) \right) \quad (5.18b)$$

$$W_L^{\Delta\Delta}(q, \omega) = \frac{f_{\pi N\Delta}^2}{m_\pi^2} \left(g'_{\Delta\Delta} \Gamma_g(t) + 2h'_{\Delta\Delta} \Gamma_h(t) + \frac{q^2}{t - m_\pi^2} \Gamma_{\pi N\Delta}^2(t) \right) \quad (5.18c)$$

$$W_T^{NN}(q, \omega) = \frac{f_{\pi NN}^2}{m_\pi^2} \left(g'_{NN} \Gamma_g(t) - h'_{NN} \Gamma_h(t) + C_{\rho NN} \frac{q^2}{t - m_\rho^2} \Gamma_{\rho NN}^2(t) \right) \quad (5.19a)$$

$$W_T^{N\Delta}(q, \omega) = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \left(g'_{N\Delta} \Gamma_g(t) - h'_{N\Delta} \Gamma_h(t) + C_{\rho N\Delta} \frac{q^2}{t - m_\rho^2} \Gamma_{\rho NN}(t) \Gamma_{\rho N\Delta}(t) \right) \quad (5.19b)$$

$$W_T^{\Delta\Delta}(q, \omega) = \frac{f_{\pi N\Delta}^2}{m_\pi^2} \left(g'_{\Delta\Delta} \Gamma_g(t) - h'_{\Delta\Delta} \Gamma_h(t) + C_{\rho \Delta\Delta} \frac{q^2}{t - m_\rho^2} \Gamma_{\rho N\Delta}^2(t) \right) \quad (5.19c)$$

where

$$t = \omega^2 - q^2, \quad C_{\rho ab} = \frac{f_{\rho ab}^2}{m_\rho^2} \left[\frac{f_{\pi ab}^2}{m_\pi^2} \right]^{-1} \quad (5.20)$$

$$\Gamma_{\pi ab}(t_\pi) = \left(\frac{\Lambda_{\pi ab}^2 - m_\pi^2}{\Lambda_{\pi ab}^2 - t} \right)^{n_{\pi ab}}, \quad \Gamma_{\rho ab}(t_\rho) = \left(\frac{\Lambda_{\rho ab}^2 - m_\rho^2}{\Lambda_{\rho ab}^2 - t} \right)^{n_{\rho ab}} \quad (5.21)$$

$$\Gamma_g(q, \omega) = \left[\frac{\Lambda_g^2}{\Lambda_g^2 - t} \right]^{n_g}, \quad \Gamma_h(q, \omega) = \frac{q^2}{m_\pi^2} \left[\frac{\Lambda_h^2}{\Lambda_h^2 - t} \right]^{n_h}, \quad (5.22)$$

with $a, b = N$ or Δ . The present program assumes

$$C_\rho \equiv C_{\rho NN} = C_{\rho N\Delta} = C_{\rho \Delta\Delta} \quad (5.23)$$

5.5 Optical potentials

5.5.1 Non-relativistic potential

It is expressed as

$$U_{\text{opt}}^N(r) = -V^N F_{\text{cr}}^N(r) - iW^N F_{\text{ci}}^N(r) - \{V_{\text{so}}^N F_{\text{sor}}^N(r) + iW_{\text{so}}^N F_{\text{soi}}^N(r)\} \mathbf{l} \cdot \mathbf{s} + V_{\text{coul}}^N(r) \quad (5.24)$$

where $N = n$ or p , and

$$F_{\text{cr}}^N(r) = \frac{1}{1 + \exp[(r - r_r^N A^{1/3})/a_r^N]} \quad (5.25)$$

$$F_{\text{ci}}^N(r) = (1 - C_{sg}^N) \frac{1}{1 + \exp[(r - r_i^N A^{1/3})/a_i^N]} + C_s^N \frac{4 \exp[(r - r_i^N A^{1/3})/a_i^N]}{(1 + \exp[(r - r_i A_{\text{tg}}^{1/3})/a_i])^2} + C_g^N \exp \left[\left(\frac{r - r_g^N A^{1/3}}{a_g^N} \right)^2 \right] \quad (5.26)$$

$$F_{\text{sor}}^{\text{N}}(r) = 2 \left(\frac{\hbar}{m_{\pi}c} \right)^2 \frac{1}{a_{\text{so}}^{\text{N}}} \frac{1}{r} \frac{\exp[(r - r_{\text{sor}}^{\text{N}} A^{1/3})/a_{\text{so}}^{\text{N}}]}{(1 + \exp[(r - r_{\text{sor}}^{\text{N}} A^{1/3})/a_{\text{so}}^{\text{N}}])^2} \quad (5.27)$$

$$F_{\text{soi}}^{\text{N}}(r) = 2 \left(\frac{\hbar}{m_{\pi}c} \right)^2 \frac{1}{a_{\text{soi}}^{\text{N}}} \frac{1}{r} \frac{\exp[(r - r_{\text{soi}}^{\text{N}} A^{1/3})/a_{\text{soi}}^{\text{N}}]}{(1 + \exp[(r - r_{\text{soi}}^{\text{N}} A^{1/3})/a_{\text{soi}}^{\text{N}}])^2} \quad (5.28)$$

$$V_{\text{coul}}^{\text{N}}(r) = \begin{cases} Z_{\text{N}} Z_{\text{tg/rs}} \frac{e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2} \right), & (r < R_c = r_c^{\text{N}} A^{1/3}) \\ Z_{\text{N}} Z_{\text{tg/rs}} \frac{e^2}{r}, & (r \geq R_c) \end{cases} \quad (5.29)$$

with

$$C_s^{\text{N}} = C_{sg}^{\text{N}}, \quad C_g^{\text{N}} = 0, \quad \text{if } r_g = 0 \quad (5.30)$$

$$C_g^{\text{N}} = C_{sg}^{\text{N}}, \quad C_s^{\text{N}} = 0, \quad \text{if } r_g \neq 0 \quad (5.31)$$

The present program uses the fixed value

$$\left(\frac{\hbar}{m_{\pi}c} \right)^2 = 2.0 \text{ [fm}^2\text{]} \quad (5.32)$$

5.5.2 Dirac phenomenogy potential

The scalar and the vector optical potentials are expressed respectively as

$$\begin{aligned} U_{\text{S}}^{\text{N}}(r) &= V_{\text{S}}^{\text{N}} F_{\text{Sr}}^{\text{N}}(r) - iW_{\text{S}}^{\text{N}} F_{\text{Si}}^{\text{N}}(r) \\ U_{\text{V}}^{\text{N}}(r) &= V_{\text{V}}^{\text{N}} F_{\text{Vr}}^{\text{N}}(r) - iW_{\text{V}}^{\text{N}} F_{\text{Vi}}^{\text{N}}(r) + V_{\text{coul}}^{\text{N}}(r) \end{aligned} \quad (5.33)$$

where N = n or p , and

$$F_{\text{Sr}}^{\text{N}}(r) = \frac{1}{1 + \exp[(r - r_{\text{Sr}}^{\text{N}} A^{1/3})/a_{\text{Sr}}^{\text{N}}]}, \quad F_{\text{Si}}^{\text{N}}(r) = \frac{1}{1 + \exp[(r - r_{\text{Si}}^{\text{N}} A^{1/3})/a_{\text{Si}}^{\text{N}}]} \quad (5.34)$$

$$F_{\text{Vr}}^{\text{N}}(r) = \frac{1}{1 + \exp[(r - r_{\text{Vr}}^{\text{N}} A^{1/3})/a_{\text{Vr}}^{\text{N}}]}, \quad F_{\text{Vi}}^{\text{N}}(r) = \frac{1}{1 + \exp[(r - r_{\text{Vi}}^{\text{N}} A^{1/3})/a_{\text{Vi}}^{\text{N}}]} \quad (5.35)$$

$$V_{\text{coul}}^{\text{N}}(r) = \begin{cases} Z_{\text{N}} Z_{\text{tg/rs}} \frac{e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2} \right), & (r < R_c = r_c^{\text{N}} A^{1/3}), \\ Z_{\text{N}} Z_{\text{tg/rs}} \frac{e^2}{r}, & (r \geq R_c) \end{cases} \quad (5.36)$$

5.6 Miscellaneous

5.6.1 Convergence check of the response functions with respect to J_{tr}

EPSJ: The criterion for the convergence check of the response functions with respect to J_{tr} . It determines the smallest J_{tr} , J_{conv} , which satisfies

$$\frac{(2J_{\text{conv}} + 1)R_{J_{\text{conv}}}(q, \omega)}{\sum^{J_{\text{conv}}} (2J_{\text{tr}} + 1)R_{J_{\text{tr}}}(q, \omega)} < \text{EPSJ} \quad (5.37)$$

5.6.2 Artificial shift of the transferred energy, ω_{shift}

In calculation of cross sections, the kinematical factors and distorted waves are calculated for the incident and exit energies $E_{i,\text{lab}}$ and $E_{f,\text{lab}} = E_{i,\text{lab}} - \omega_{\text{lab}}$, but we use the response functions

$$R(r', r; \omega - \omega_{\text{shift}})$$

instead of $R(r', r; \omega)$

6 Examples

6.1 Quasi-Elastic Scattering (QES)

[Example] $^{12}\text{C}(p, n)$ at $T_{p,\text{lab}}=345\text{MeV}$, $\theta_{\text{lab}}=16, 22\text{deg}$, $\omega=20-100\text{MeV}$

RPA with Δ

Mean field parameters for the nucleons

Shape parameters: from ref.[15]

Spin-orbit potential: from ref.[6]

The Perey factor parameters: from ref.[6]

Spreading widths: both for particles and holes by the empirical formula [14].

Fermi Energy: Calculated by the given potential

Mean field parameters for Delta: from ref.[6]

Effective ph interaction : $V_t + \text{LM} + \text{OBEP}$.

Spin-scalar interaction parameter : from ref.[17]

OBEP parameters: from ref.[6]

LM parameters : $(g'_{\text{NN}}, g'_{\text{N}\Delta}, g'_{\Delta\Delta}) = (0.6, 0.3, 0.5)$ without form factor [6]

DWIA with full amplitudes

NN-t-matrix : Bugg-Wilkin [12] in terms of the optimal factorization method.

Optical potentials : the global one by Cooper et al. [10] both for proton and neutron.

[Input] See the file /input/C/C-QES.dat

1121201701

C(p,n) Tp=345 th=16,22 w=20-100 Wp=spw Wh=spw RPA wDelta DWIA BugWil Coop-Coop

12110011

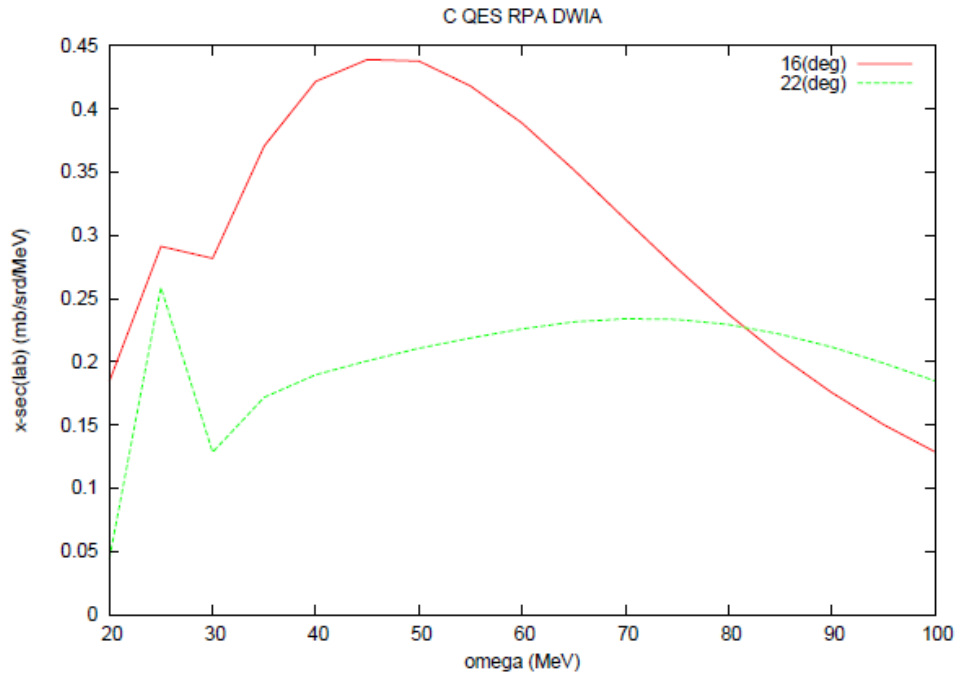
1000000111

1.0	12.0	6.0	1.0	0.0			
2.0	345.0	2.0	16.0	6.0	17.0	20.0	5.0
3.0	0.0	10.0	1.0	2.0			
4.0	10.0	50.0	3.0	120.0			
5.1	0.0	1.27	0.67	6.5	1.27	0.67	1.27
5.2	0.0	1.27	0.67	6.5	1.27	0.67	1.27
5.3	0.0	1.27	0.67	6.5	1.27	0.67	1.27
5.4	0.0	1.27	0.67	6.5	1.27	0.67	1.27
5.5	30.0	1.27	0.67	0.0	1.27	0.67	1.27
6.3	999.0	1.27	0.67	1.0	0.0	1.27	0.67
6.4	999.0	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	18.0	110.0	16.559
7.2	1.27	0.67	0.3	10.75	18.0	110.0	13.818
7.3	1.27	0.67	0.3	10.75	18.0	110.0	16.559
7.4	1.27	0.67	0.3	10.75	18.0	110.0	13.818
8.1	2.0						
0	1	1.5	18.720				
0	0	0.5					
8.2	2.0						
0	1	1.5	15.956				
0	0	0.5					

(continued)

9.0	283.0					
9.1	0.080	2.0	2.18			
9.2	139.0	0.0	1300.0	1.0	1200.0	1.0
9.3	770.0	0.0	2000.0	1.0	2000.0	1.0
9.4	0.6	0.3	0.5			
10.0	0.0	40.0	3.0	3.0	3.0	3.0
0.0						

[Output] See the file /OUTPUT/C/C.QES_RPA_DWIA.log
The calculated cross sections are shown in the figure below.



6.2 1+ mode (GT + IVSM + etc.) excitation

[**Exsample 1**] No correlation (0th) and PWIA

Zr(p, n)Nb(1^+) at $T_{p,\text{lab}} = 295\text{MeV}$, $\theta_{\text{lab}} = 0\text{deg}$, $\omega = 0-30\text{MeV}$

No correlation (0th)

Mean field parameters

Shape parameters: from ref.[15]

Perey factor parameters: from ref.[6]

Imaginary potential depth of particles: 0.5MeV

Spreading width of holes: 0.0MeV

PWIA with full amplitudes

NN-t-matrix : Franey-Love NN-t-matrix [9]

[**Input**] See the file /input/Zr/Zr_pn_1+_0th_PWIA.dat

1226201701

Zr(p,n)1+ 295MeV 0deg w=0-30 Wp=0.5 wh=0 EF=exp 0th PWIA FL

00001000

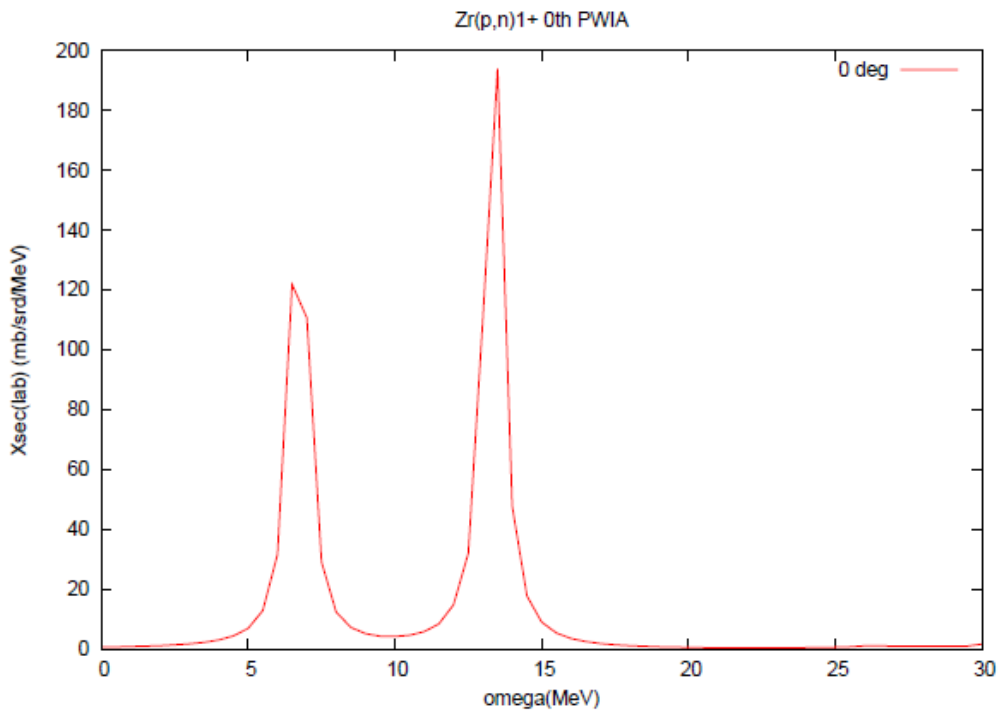
1000000001

1.0	90.0	40.0	1.0	0.0			
2.0	295.0	1.0	0.0	0.0	61.0	0.0	0.5
3.0	1.0	1.0	2.0	2.0			
4.0	16.0	80.0	3.0	120.0			
5.1	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.2	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.3	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.4	0.0	1.27	0.67	8.24	1.27	0.67	1.27
6.3	0.5	1.27	0.67	1.0	0.0	1.27	0.67
6.4	0.5	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.2	1.27	0.67	0.3	10.75	18.0	110.0	6.758
7.3	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.4	1.27	0.67	0.3	10.75	18.0	110.0	6.758
8.1	11.0						
0	4	4.5	11.97				
1	1	0.5					
1	1	1.5					
0	3	2.5					
0	3	3.5					
0	2	1.5					
1	0	0.5					
0	2	2.5					
0	1	0.5					
0	1	1.5					
0	0	0.5					

(continued)

8.2	10.0		
1	1	0.5	8.35
1	1	1.5	
0	3	2.5	
0	3	3.5	
0	2	1.5	
1	0	0.5	
0	2	2.5	
0	1	0.5	
0	1	1.5	
0	0	0.5	
10.0	0.0	80.0	
0.0			

[Output] See the file /OUTPUT/Zr/Zr_pn_1+_0th_PWIA.log
The calculated cross sections at 0 deg are shown in the figure below.



[Example 2] RPA without Δ and DWIA with full amplitudes

Zr(p, n)Nb(1^+) at $T_{p,lab} = 295\text{MeV}$, $\theta_{lab} = 0, 2\text{deg}$, $\omega = 0-30\text{MeV}$

RPA without Δ

Mean field parameters

Shape parameters: from ref.[15]

Perey factor parameters: from ref.[6]

Imaginary potential depth of particles: calculated from the empirical formula [14]

Spreading width of holes: calculated from the empirical formula [14]

Fermi energy: experimental values

Effective ph interaction: LM + OBEP.

OBEP parameters: from ref.[16]

LM parameters: $g'_{NN} = 0.6$ without form factor[7].

DWIA with full amplitudes

NN-t-matrix : Franey-Love NN-t-matrix [9]

Optical potential :

for proton, the global one by Cooper et al. [10]

for neutron the global one by Shen et al. [11]

[Input] See the file /input/Zr/Zr_pn_1+_RPA_noD_DWIA.dat

1226201701

Zr(p,n)1+ 295MeV 0,2deg w=0-30 Wp=spw Wh=spw RPA noD DWIA FL C-S

12001011

1

1.0	90.0	40.0	1.0	0.0			
2.0	295.0	2.0	0.0	2.0	61.0	0.0	0.5
3.0	1.0	1.0	2.0	2.0			
4.0	16.0	80.0	3.0	120.0			
5.1	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.2	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.3	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.4	0.0	1.27	0.67	8.24	1.27	0.67	1.27
6.3	999.0	1.27	0.67	1.0	0.0	1.27	0.67
6.4	999.0	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.2	1.27	0.67	0.3	10.75	18.0	110.0	6.758
7.3	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.4	1.27	0.67	0.3	10.75	18.0	110.0	6.758
8.1	11.0						
0	4	4.5	11.97				
1	1	0.5					
1	1	1.5					
0	3	2.5					
0	3	3.5					
0	2	1.5					
1	0	0.5					

(continued)

0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
8.2	10.0					
1	1	0.5	8.35			
1	1	1.5				
0	3	2.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
9.1	0.0778	1.69706	2.94058			
9.2	138.03	0.0	1300.0	1.0	1200.0	1.0
9.3	769.0	0.0	1400.0	1.0	1400.0	2.0
9.4	0.6					
10.0	0.0	80.0	3.0	6.0	4.0	
0.0						

[Output] See the file /OUTPUT/Zr/Zr_pn_1+_RPA_noD_DWIA.log

[Exsample 3] RPA for N but TDA for Δ , and DWIA with full amplitudes

Zr(p, n)Nb(1^+) at $T_{p,lab} = 295\text{MeV}$, $\theta_{lab} = 0, 2\text{deg}$, $\omega = 0-30\text{MeV}$

RPA for N-space and TDA for N- Δ coupling

Mean field paremeters for the nucleons

Shape parameters: from ref.[15]

Perey factor parameters: from ref.[6]

Imaginary potential depth of particles: calculated from the emperical formula [14]

Spreading width of holes: calculated from the emperical formula [14]

Fermi energy: experimental values

Mean field parameters for Delta: from ref.[6]

Effective ph interaction : LM + OBEP.

OBEP parameters: from ref.[16]

LM parameters : $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.35, 0.5)$ without form factor [7]

DWIA with full amplitudes

NN-t-matrix : Franey-Love NN-t-matrix [9]

Optical potential :

for proton, the global one by Cooper et al. [10]

for neutron the global one by Shen et al. [11]

[Input] See the file /input/Zr/Zr_pn_1+_RPA_DWIA.dat

1226201701

Zr(p,n)1+ 295MeV 0,2deg w=0-30 Wp=spw Wh=spw RPA+TDA(D) DWIA FL CS

13101011

1

1.0	90.0	40.0	1.0	0.0			
2.0	295.0	2.0	0.0	2.0	61.0	0.0	0.5
3.0	1.0	1.0	2.0	2.0			
4.0	16.0	80.0	3.0	120.0			
5.1	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.2	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.3	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.4	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.5	30.0	1.27	0.67	0.0	1.27	0.67	1.27
6.3	999.0	1.27	0.67	1.0	0.0	1.27	0.67
6.4	999.0	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.2	1.27	0.67	0.3	10.75	18.0	110.0	6.758
7.3	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.4	1.27	0.67	0.3	10.75	18.0	110.0	6.758

8.1 11.0

0 4 4.5 11.97

1 1 0.5

1 1 1.5

0 3 2.5

0 3 3.5

0 2 1.5

1 0 0.5

0 2 2.5

0 1 0.5

0 1 1.5

0 0 0.5

8.2 10.0

1 1 0.5 8.35

1 1 1.5

0 3 2.5

0 3 3.5

0 2 1.5

1 0 0.5

0 2 2.5

0 1 0.5

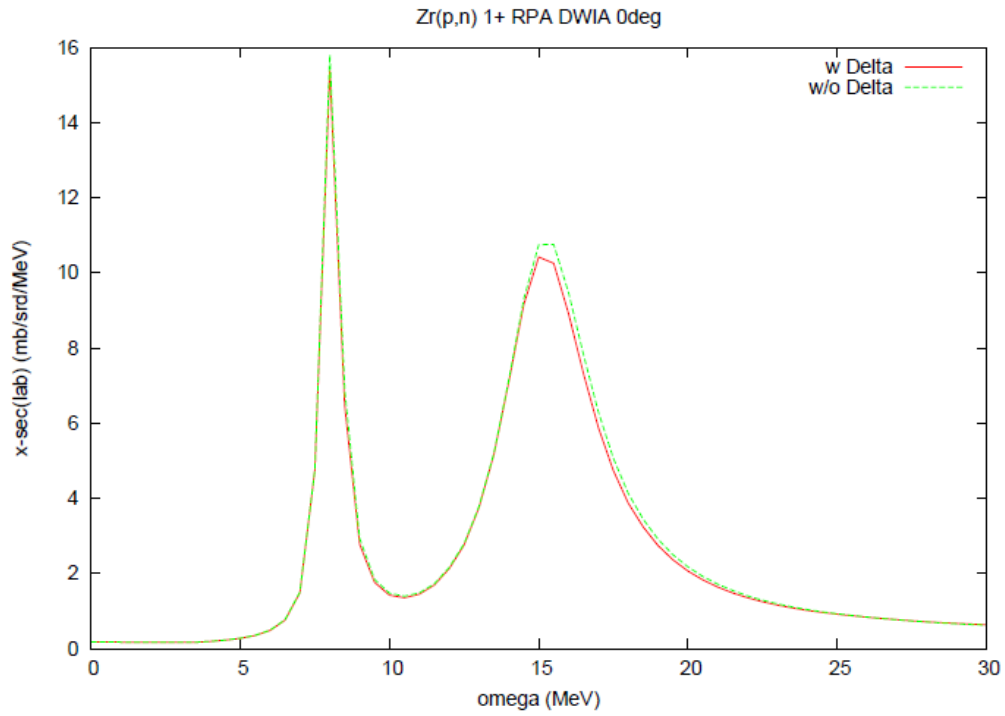
0 1 1.5

0 0 0.5

(Continued)

9.1	0.0778	1.69706	2.94058			
9.2	138.03	0.0	1300.0	1.0	1200.0	1.0
9.3	769.0	0.0	1400.0	1.0	1400.0	2.0
9.4	0.6	0.35	0.5			
10.0	0.0	80.0	3.0	6.0	4.0	
0.0						

[Output] See the file /OUTPUT/Zr/Zr.pn.1+_RPA.DWIA.log
The calculated cross sections of ex. 2 and 3. at 0 deg are shown in the figure below.



[Exsample 4] RPA for N but TDA for Δ , and DWIA with only C+T amplitudes

Zr(p, n)Nb(1^+) at $T_{p,lab} = 295\text{MeV}$, $\theta_{lab} = 0, 2\text{deg}$, $\omega = 0-30\text{MeV}$

RPA for N-space and TDA for N- Δ coupling

Mean field parameters for the nucleons

Shape parameters: from ref.[15]

Perey factor parameters: from ref.[6]

Imaginary potential depth of particles: calculated from the emperical formula [14]

Spreading width of holes: calculated from the emperical formula [14]

Fermi energy: experimental values

Mean field parameters for Delta: from ref.[6]

Effective ph interaction : LM + OBEP.

OBEP parameters: from ref.[16]

LM parameters : $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.35, 0.5)$ without form factor [7]

DWIA with only central+ tensor NN amplitudes

NN-t-matrix : Franey-Love NN-t-matrix [9]

Optical potential :

for proton, the global one by Cooper et al. [10]

for neutron the global one by Shen et al. [11]

[Input] See the file /input/Zr/Zr_pn_1+_RPA_DWIA_C+T.dat

1121201702

Zr(p,n)1+ 295MeV 0,2deg w5-25 Wp=spw wh=spw RPA+TDA(D) DWIA FL(C+T) C-S

13101711

1

1.0	90.0	40.0	1.0	0.0			
2.0	295.0	2.0	0.0	2.0	61.0	0.0	0.5
3.0	1.0	1.0	2.0	2.0			
4.0	16.0	80.0	3.0	120.0			
5.1	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.2	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.3	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.4	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.5	30.0	1.27	0.67	0.0	1.27	0.67	1.27
6.3	999.0	1.27	0.67	1.0	0.0	1.27	0.67
6.4	999.0	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.2	1.27	0.67	0.3	10.75	18.0	110.0	6.758
7.3	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.4	1.27	0.67	0.3	10.75	18.0	110.0	6.758

(continued)

8.1	11.0					
0	4	4.5	11.97			
1	1	0.5				
1	1	1.5				
0	3	2.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
8.2	10.0					
1	1	0.5	8.35			
1	1	1.5				
0	3	2.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
9.1	0.0778	1.69706	2.94058			
9.2	138.03	0.0	1300.0	1.0	1200.0	1.0
9.3	769.0	0.0	1400.0	1.0	1400.0	2.0
9.4	0.6	0.35	0.5			
10.0	0.0	80.0	3.0	6.0	4.0	
0.0						

[Output] See the file /OUTPUT/Zr/Zr_pn.1+_RPA.DWIA-C+T.log

6.3 A spin-dipole excitation

[Exsample] $^{208}\text{Pb}(p, n)\text{Bi}(1^-)$ at $T_{p,\text{lab}} = 296\text{MeV}$, $\theta_{\text{lab}} = 0$, $2\omega = 10-50\text{MeV}$

RPA in N-space while TDA for N- Δ coupling

Mean field parameters for the nucleons

Shape parameters: from ref.[15]

Spin-orbit potential: from ref.[17]

The Perey factor parameters: from ref.[17]

Spreading widths: both for particles and holes by the emperical formula [17].

Mean field parameters for Delta: from ref.[17]

Effective ph interaction : $V_t + \text{LM} + \text{OBEP}$.

Spin-scalar interaction parameter : from ref.[17]

OBEP parameters: from ref.[16]

LM parameters : $(g'_{\text{NN}}, g'_{\text{N}\Delta}, g'_{\Delta\Delta}) = (0.64, 0.35, 0.5)$ without form factor [17]

DWIA with full amplitudes

NN-t-matrix : Franey-Love NN-t-matrix [9]

Optical potential :

for proton, the global one by Cooper et al. [10]

for neutron the global one by Shen et al. [11]

[Input] See the file /input/Pb/Pb_pn_1-_RPA_DWIA.dat

1230201701

Pb(p,n)1- Tp=296 th=0,2 w=10-50 Wp=spw Wh=spw RPA+TDA(D), DWIA FL C-S

13101011

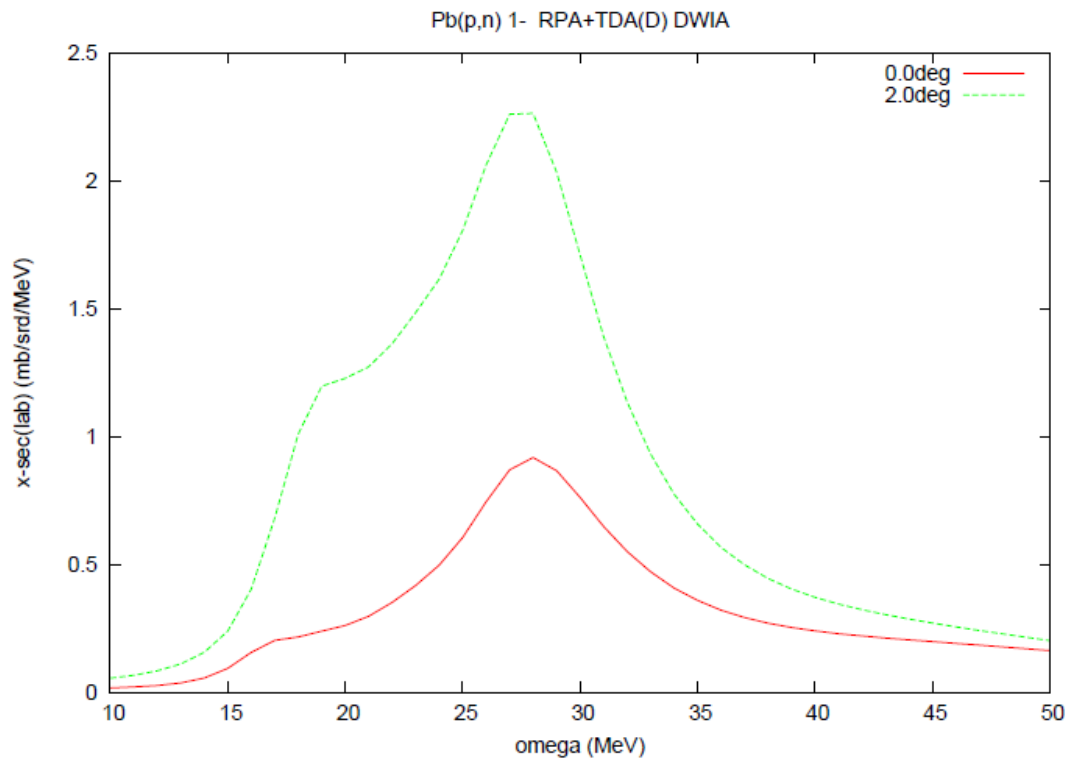
1000000101

1.0	208.0	82.0	1.0	0.0			
2.0	296.0	2.0	0.0	2.0	41.0	10.0	1.0
3.0	1.0	1.0	1.0	1.0			
4.0	16.0	80.0	3.0	120.0			
5.1	0.0	1.27	0.67	7.50	1.27	0.67	1.27
5.2	0.0	1.27	0.67	7.50	1.27	0.67	1.27
5.3	0.0	1.27	0.67	7.50	1.27	0.67	1.27
5.4	0.0	1.27	0.67	7.50	1.27	0.67	1.27
5.5	30.0	1.27	0.67	0.0	1.27	0.67	1.27
6.3	999.0	1.27	0.67	1.0	0.0	1.27	0.67
6.4	999.0	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	26.0	110.0	5.659
7.2	1.27	0.67	0.3	10.75	26.0	110.0	5.902
7.3	1.27	0.67	0.3	10.75	26.0	110.0	5.659
7.4	1.27	0.67	0.3	10.75	26.0	110.0	5.902
8.1	22.0						
2	1	0.5	7.368				
1	3	2.5					
2	1	1.5					
0	6	6.5					
0	5	4.5					
1	3	3.5					
1	2	1.5					

(continued)

2	0	0.5				
0	5	5.5				
0	4	3.5				
1	2	2.5				
0	4	4.5				
1	1	0.5				
0	3	2.5				
1	1	1.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
8.2	16.0					
2	0	0.5	8.008			
1	2	1.5				
0	5	5.5				
0	4	3.5				
1	2	2.5				
0	4	4.5				
1	1	0.5				
0	3	2.5				
1	1	1.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
9.0	283.0					
9.1	0.0778	1.69706	2.94058			
9.2	138.03	0.0	1300.0	1.0	1200.0	1.0
9.3	769.0	0.0	1400.0	1.0	1400.0	2.0
9.4	0.64	0.35	0.5			
10.0	0.0	80.0	3.0	7.0	4.0	
0.0						

[Output] See the file /OUTPUT/Pb/Pb_pn.1-_RPA+TDA_DWIA.log
The calculated cross sections are shown in the figure below.



7 Definition of output quantities

7.1 Cross sections

The T-matrix for the reaction (2.1) in the incident nucleon-target (NA) c.m. system is specified as

$$T_{n0}^{fi} = \langle \mathbf{k}_f m_{s_f} N', \Phi_n | T | \mathbf{k}_i m_{s_i} N, \Phi_0 \rangle \quad (7.1)$$

where \mathbf{k}_i and \mathbf{k}_f represent the incident and exit momenta, m_{s_i} and m_{s_f} do the spin projections of N and N' , and i and f denote the sets of the quantum numbers, $(\mathbf{k}_i, m_{s_i}, N)$ and $(\mathbf{k}_f, m_{s_f}, N')$, respectively. The state vectors Φ_0 and Φ_n are those of the ground state of the target A and the n-th state of the residual nuclear system X_n , respectively.

The unpolarized double differential cross section in the c.m. system is expressed as

$$\begin{aligned} I_{\text{cm}}(\theta_{\text{cm}}, \omega_{\text{cm}}) &= \frac{d^2\sigma}{d\Omega_{\text{cm}} d\omega_{\text{cm}}} \\ &= \frac{\mu_i \mu_f k_f}{(2\pi)^2 k_i} \sum_{m_{s_f}} \frac{1}{2} \sum_{m_{s_i}} \sum_n \left| T_{n0}^{fi} \right|^2 \delta(\omega_{\text{cm}} - (E_{X_n} - E_A)) \end{aligned} \quad (7.2)$$

where θ_{cm} is the scattering angle and ω_{cm} is the transferred energy to the target

$$\omega_{\text{cm}} = E_N - E_{N'} \quad (7.3)$$

in the NA c.m. system. The relativistic reduced energies μ_i and μ_f are given by

$$\mu_i = \frac{E_N E_A}{E_{\text{total}}}, \quad \mu_f = \frac{E_{N'} E_{X_n}}{E_{\text{total}}}, \quad (E_{\text{total}} = E_N + E_A) \quad (7.4)$$

where E_N , $E_{N'}$, E_A and E_{X_n} are the relativistic energies of N, N' , A and X_n , respectively, in the NA c.m. system, namely

$$E_a = \sqrt{m_a^2 + k_a^2} \quad (7.5)$$

We introduce the cross section per unit excitation energy as

$$\frac{d^2\sigma}{d\Omega_{\text{cm}} d\omega} = \frac{\mu_i \mu_f k_f}{(2\pi)^2 k_i} \frac{1}{2} \sum_{m_{s_i} m_{s_f}} \sum_n \left| T_{n0}^{fi} \right|^2 \delta(\omega - (\mathcal{E}_n - \mathcal{E}_0)) \quad (7.6)$$

where \mathcal{E}_0 and \mathcal{E}_n are the intrinsic energies of the states Φ_0 and Φ_n , respectively, and

$$\omega = \mathcal{E}_n - \mathcal{E}_0 \quad (7.7)$$

is the excitation energy with respect to the target ground state.

Then, the unpolarized double differential cross section in the c.m. system is given as

$$I_{\text{cm}}(\theta_{\text{cm}}, \omega_{\text{cm}}) = \frac{d\omega}{d\omega_{\text{cm}}} \frac{d^2\sigma}{d\Omega_{\text{cm}} d\omega} = \frac{K}{2} \sum_{m_{s_f}, m_{s_i}} \sum_n \left| T_{n0}^{fi} \right|^2 \delta(\omega - (\mathcal{E}_n - \mathcal{E}_0)) \quad (7.8)$$

with the kinematical factor

$$K = \frac{\mu_i \mu_f k_f}{(2\pi)^2 k_i} \frac{d\omega}{d\omega_{\text{cm}}} = \frac{\mu_i \mu_f k_f E_{\text{total}}}{(2\pi)^2 k_i \mathcal{E}_n} \quad (7.9)$$

We introduce the notations, Tr , for the trace over the nucleon spin projections

$$\text{Tr}[AB] = \sum_{m_s m'_s} \langle m_s | A | m'_s \rangle \langle m'_s | B | m_s \rangle \quad (7.10)$$

and the notation, Tr' , for the sum over the final nuclear states Φ_n , that is,

$$\text{Tr}' [TT^\dagger] \equiv \sum_n \langle \Phi_n | T | \Phi_0 \rangle \langle \Phi_0 | T^\dagger | \Phi_n \rangle \delta(\omega - (\mathcal{E}_n - \mathcal{E}_0)) \quad (7.11)$$

then the cross section can be written as

$$I_{\text{cm}}(\theta_{\text{cm}}, \omega_{\text{cm}}) = \frac{K}{2} \text{Tr} \text{Tr}' (TT^\dagger) \quad (7.12)$$

The unpolarized double differential cross section in the laboratory frame is given by

$$I_{\text{lab}}(\theta_{\text{lab}}, \omega_{\text{lab}}) = \frac{k_{f,\text{lab}}}{k_f} I_{\text{cm}}(\theta_{\text{cm}}, \omega_{\text{cm}}) \quad (7.13)$$

where θ_{lab} , ω_{lab} and $\mathbf{k}_{f,\text{lab}}$ are the scattering angle, the transferred energy and the exit nucleon momentum in the laboratory frame, respectively.

7.2 Spin observables

7.2.1 Coordinate systems

The following coordinate systems are used to present the spin observables.

(1) $[\mathbf{S}, \mathbf{N}, \mathbf{L}]$, and $[\mathbf{S}', \mathbf{N}', \mathbf{L}']$ systems

They are defined as

$$\hat{\mathbf{L}} = \frac{\mathbf{k}_{i,\text{lab}}}{|\mathbf{k}_{i,\text{lab}}|}, \quad \hat{\mathbf{N}} = \frac{\mathbf{k}_{i,\text{lab}} \times \mathbf{k}_{f,\text{lab}}}{|\mathbf{k}_{i,\text{lab}} \times \mathbf{k}_{f,\text{lab}}|}, \quad \hat{\mathbf{S}} = \hat{\mathbf{N}} \times \hat{\mathbf{L}} \quad (7.14)$$

$$\hat{\mathbf{L}}' = \frac{\mathbf{k}_{f,\text{lab}}}{|\mathbf{k}_{f,\text{lab}}|}, \quad \hat{\mathbf{N}}' = \hat{\mathbf{N}}, \quad \hat{\mathbf{S}}' = \hat{\mathbf{N}}' \times \hat{\mathbf{L}}' \quad (7.15)$$

where $\mathbf{k}_{i,\text{lab}}$ is the incident nucleon momentum in the lab. frame. Their directions are denoted by L, N, S and L', N', S' , respectively. The reaction normal $\hat{\mathbf{N}}$ is also called the y-direction.

(2) $[\mathbf{q}, \mathbf{n}, \mathbf{p}]$ system

It is defined as

$$\hat{\mathbf{q}} = \frac{\mathbf{q}_{\text{cm}}}{|\mathbf{q}_{\text{cm}}|}, \quad \hat{\mathbf{n}} = \hat{\mathbf{N}}, \quad \hat{\mathbf{p}} = \hat{\mathbf{q}} \times \hat{\mathbf{n}}, \quad (7.16)$$

where $\mathbf{q}_{\text{cm}} = \mathbf{k}_f - \mathbf{k}_i$ is the momentum transfer in the c.m. system. Their directions are denoted by q, n, p .

7.2.2 Spin observables

The polarization P_y , the analyzing power A_y and the polarization transfer coefficients D_{ij} are given by

$$P_y = \frac{\text{TrTr}'[TT^\dagger \sigma_{0y}]}{\text{TrTr}'[TT^\dagger]}, \quad A_y = \frac{\text{TrTr}'[T \sigma_{0y} T^\dagger]}{\text{TrTr}'[TT^\dagger]}, \quad D_{ij} = \frac{\text{TrTr}'[T \sigma_{0j} T^\dagger \sigma_{0i}]}{\text{TrTr}'[TT^\dagger]}. \quad (7.17)$$

where σ_{0i} is the Pauli spin matrix of the direction i for the incident (exit) nucleon.

Note the order of ij in the definition of D_{ij} . Some literatures have the opposite order.

The program outputs $P_y, A_y, (D_{SS}, D_{SL}, D_{NN}, D_{LS}, D_{LL})$ and $(D_{qq}, D_{qp}, D_{nn}, D_{pq}, D_{pp})$.

7.2.3 Polarized cross sections

We decompose the NA T-matrix into

$$T = T_0 + T_n \sigma_{0n} + T_q \sigma_{0q} + T_p \sigma_{0p} \quad (7.18)$$

where σ 's are the Pauli spin matrices for the scattering nucleon (N, N'), and define the polarized cross sections $I_{\text{cm}} D_i$, which exclusively extract T_i as

$$\begin{aligned} I_{\text{cm}} D_0 &= \frac{I_{\text{cm}}}{4} [1 + D_{nn} + D_{qq} + D_{pp}] = K \text{Tr}'[T_0 T_0^\dagger] , \\ I_{\text{cm}} D_n &= \frac{I_{\text{cm}}}{4} [1 + D_{nn} - D_{qq} - D_{pp}] = K \text{Tr}'[T_n T_n^\dagger] , \\ I_{\text{cm}} D_q &= \frac{I_{\text{cm}}}{4} [1 - D_{nn} + D_{qq} - D_{pp}] = K \text{Tr}'[T_q T_q^\dagger] , \\ I_{\text{cm}} D_p &= \frac{I_{\text{cm}}}{4} [1 - D_{nn} - D_{qq} + D_{pp}] = K \text{Tr}'[T_p T_p^\dagger] , \end{aligned} \quad (7.19)$$

Following the convention, the program outputs the cross sections

$$I_{\text{lab}} D_0, \quad I_{\text{lab}} D_n, \quad I_{\text{lab}} D_q, \quad I_{\text{lab}} D_p \quad (7.20)$$

instead of $I_{\text{cm}} D_i$'s because $I_{\text{lab}} D_0 + I_{\text{lab}} D_n + I_{\text{lab}} D_q + I_{\text{lab}} D_p = I_{\text{lab}}$

7.3 NN t-matrix and Spin-longitudinal -transverse ratio

The isovector part of the NN t-matrix in the NA c.m. system is expressed as

$$\begin{aligned} t_{k,\text{NN/A}}^{(0)}(\mathbf{q}) &= \bar{J} \boldsymbol{\tau}_0 \cdot \boldsymbol{\tau}_k \{ A_1 \mathbf{1}_0 \mathbf{1}_k + B_1 (\boldsymbol{\sigma}_0 \cdot \hat{\mathbf{n}}) (\boldsymbol{\sigma}_k \cdot \hat{\mathbf{n}}) + C_1 ((\boldsymbol{\sigma}_0 \cdot \hat{\mathbf{n}}) \mathbf{1}_k + \mathbf{1}_0 (\boldsymbol{\sigma}_k \cdot \hat{\mathbf{n}})) \\ &\quad + E_1 (\boldsymbol{\sigma}_0 \cdot \hat{\mathbf{q}}) (\boldsymbol{\sigma}_k \cdot \hat{\mathbf{q}}) + F_1 (\boldsymbol{\sigma}_0 \cdot \hat{\mathbf{p}}) (\boldsymbol{\sigma}_k \cdot \hat{\mathbf{p}}) \} \end{aligned} \quad (7.21)$$

where \bar{J} is the Jacobian of the frame transformation.

The program has an option to output the absolute square of these amplitudes

$$|\bar{J} A_1(q_{\text{cm}})|^2, \quad |\bar{J} B_1(q_{\text{cm}})|^2, \quad |\bar{J} C_1(q_{\text{cm}})|^2, \quad |\bar{J} E_1(q_{\text{cm}})|^2, \quad |\bar{J} F_1(q_{\text{cm}})|^2 \quad (7.22)$$

CRDW also has an option to outputs the spin-longitudinal-transverse ratio L/T, which is defined as

$$\frac{L}{T} = \frac{D_q(\theta_{\text{cm}}, \omega_{\text{cm}})}{|E_1(q_{\text{cm}})|^2} \frac{|F_1(q_{\text{cm}})|^2}{D_p(\theta_{\text{cm}}, \omega_{\text{cm}})} \quad (7.23)$$

7.4 Response functions for the isovector operators

This program treats only the isovector transitions.

7.4.1 Spherical tensor representation of the spin and isospin operators

We use the spherical tensor representation for the nucleon spin operators $\boldsymbol{\sigma}$ as

$$\sigma_{\pm 1} = \mp \frac{\sigma_x \pm i \sigma_y}{\sqrt{2}}, \quad \sigma_0 = \sigma_z \quad (7.24)$$

and the spin transition operators \mathbf{S} from N to Δ as

$$S_{\pm 1} = \mp \frac{S_x \pm iS_y}{\sqrt{2}}, \quad S_0 = S_z \quad (7.25)$$

Similarly the nucleon isospin operators $\boldsymbol{\tau}$ and the N- Δ transition isospin operators \mathbf{T} are expressed as

$$\tau_{\pm 1} = \mp \frac{\tau_x \pm i\tau_y}{\sqrt{2}}, \quad \tau_0 = \tau_z, \quad T_{\pm 1} = \mp \frac{T_x \pm iT_y}{\sqrt{2}}, \quad T_0 = T_z \quad (7.26)$$

We note that the commonly used t_{\pm} are expressed as

$$t_{\pm} = t_x \pm it_y = \mp \frac{\tau_{\pm 1}}{\sqrt{2}} \quad (7.27)$$

We use the notations for the hermite conjugate operators as

$$S_{\pm 1}^{\dagger} = (S^{\dagger})_{\pm 1} = \mp \frac{S_x^{\dagger} \pm iS_y^{\dagger}}{\sqrt{2}} = \mp \left(\frac{S_x \mp iS_y}{\sqrt{2}} \right)^{\dagger} = -(S_{\mp 1})^{\dagger} \quad (7.28)$$

and the corresponding notations for the isospin transition operators.

Note the relations

$$S_{\mu}^{\dagger} = (-1)^{\mu} (S_{-\mu})^{\dagger}, \quad T_{\nu}^{\dagger} = (-1)^{\nu} (T_{-\nu})^{\dagger} \quad (7.29)$$

7.4.2 One-body transition operators

Noting $\sum_i^A \mathbf{r}_i = 0$, we introduce the position vector \mathbf{r}'_i

$$\mathbf{r}'_i = \mathbf{r}_i - \frac{1}{A-1} \sum_{j \neq i} \mathbf{r}_j = \frac{A}{A-1} \mathbf{r}_i \quad (7.30)$$

and the intrinsic momentum transfer

$$\mathbf{q}' = \frac{A-1}{A} \mathbf{q}_{\text{cm}} \quad (7.31)$$

We consider the following one-body transition operators from N to N and from N to Δ .

(1) Isovector spin-scalar operators

$$O_{\nu, S}(\mathbf{q}') \equiv \frac{1}{\sqrt{2}} \sum_k \tau_{\nu, k} e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.32)$$

For this mode Δ does not contribute. The weak Fermi transition operator is given by

$$O_{\pm 1, S}(\mathbf{q}' = 0) = \sum_k t_{\pm, k} \quad (7.33)$$

(2) Isovector spin-vector operators

$$O_{\nu\mu}^N(\mathbf{q}') \equiv \frac{1}{\sqrt{2}} \sum_k \tau_{\nu, k} \sigma_{\mu, k} e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.34)$$

$$O_{\nu\mu}^{\Delta}(\mathbf{q}') \equiv \frac{1}{\sqrt{2}} \sum_k \left\{ T_{\nu, k} S_{\mu, k} + T_{\nu, k}^{\dagger} S_{\mu, k}^{\dagger} \right\} e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.35)$$

The weak Gamow-Teller transition operator is given by

$$O_{\pm 1, \mu}(\mathbf{q}' = 0) = \sum_k t_{\pm, k} \sigma_{\mu, k} \quad (7.36)$$

(3) **Isovector spin-longitudinal operators**

$$O_{\nu, L}^N(\mathbf{q}') \equiv -\frac{1}{\sqrt{2}} \sum_k \tau_{\nu, k} (\boldsymbol{\sigma}_k \cdot \hat{\mathbf{q}}') e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.37)$$

$$O_{\nu, L}^\Delta(\mathbf{q}') \equiv -\frac{1}{\sqrt{2}} \sum_k \left\{ T_{\nu, k} (\mathbf{S}_k \cdot \hat{\mathbf{q}}') + T_{\nu, k}^\dagger (\mathbf{S}_k^\dagger \cdot \hat{\mathbf{q}}') \right\} e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.38)$$

(4) **Isovector spin-transverse operators**

$$O_{\nu\mu, T}^N(\mathbf{q}') \equiv -\frac{1}{2} \sum_k \tau_{\nu, k} [\boldsymbol{\sigma}_k \times \mathbf{q}']_\mu e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.39)$$

$$O_{\nu\mu, T}^\Delta(\mathbf{q}') \equiv -\frac{1}{2} \sum_k \left\{ T_{\nu, k} [\mathbf{S}_k \times \mathbf{q}']_\mu + T_{\nu, k}^\dagger [\mathbf{S}_k^\dagger \times \mathbf{q}']_\mu \right\} e^{-i\mathbf{q}' \cdot \mathbf{r}'_k} \quad (7.40)$$

7.4.3 Isovector response functions

Corresponding to the above one-body operators, the program calculates the following response functions in RPA, TDA or the mean field approximation (0-th).

We restrict the spin of the target to be $J_0 = 0$ and write the excitation energy as

$$\omega_n = \mathcal{E}_n - \mathcal{E}_0 \quad (7.41)$$

(1) **Isovector spin-scalar response function**

$$R_{\nu, S}(q', \omega) \equiv \sum_{n \neq 0} |\langle \Phi_n | O_{\nu, S}(\mathbf{q}') | \Phi_0 \rangle|^2 \delta(\omega - \omega_n) \quad (7.42)$$

(2) **Isovector spin-vector response function**

$$R_{\nu, V}^{ab}(q', \omega) \equiv \sum_{n \neq 0} \sum_\mu \langle \Phi_0 | [O_{\nu\mu}^a(\mathbf{q}')]^\dagger | \Phi_n \rangle \langle \Phi_n | O_{\nu\mu}^b(\mathbf{q}') | \Phi_0 \rangle \delta(\omega - \omega_n) \quad (7.43)$$

(3) **Isovector spin-longitudinal response function**

$$R_{\nu, L}^{ab}(q', \omega) \equiv \sum_{n \neq 0} \langle \Phi_0 | [O_{\nu, L}^a(\mathbf{q}')]^\dagger | \Phi_n \rangle \langle \Phi_n | O_{\nu, L}^b(\mathbf{q}') | \Phi_0 \rangle \delta(\omega - \omega_n) \quad (7.44)$$

(4) **Isovector spin-transverse response function**

$$R_{\nu, T}^{ab}(q', \omega) \equiv \sum_{n \neq 0} \langle \Phi_0 | [O_{\nu, T}^a(\mathbf{q}')]^\dagger | \Phi_n \rangle \langle \Phi_n | O_{\nu, T}^b(\mathbf{q}') | \Phi_0 \rangle \delta(\omega - \omega_n) \quad (7.45)$$

In the above $a, b = N$, or Δ . Since $J_0 = 0$, all these response functions are independent of the direction of \mathbf{q}' .

We utilize the relation

$$R_{\nu, V}^{ab}(q', \omega) = 2R_{\nu, T}^{ab}(q', \omega) + R_{\nu, L}^{ab}(q', \omega) \quad (7.46)$$

to get the spin-transverse response functions. It can be proved by the vector relation

$$(\mathbf{A} \cdot \hat{\mathbf{q}})(\mathbf{B} \cdot \hat{\mathbf{q}}) + [\mathbf{A} \times \hat{\mathbf{q}}] \cdot [\mathbf{B} \times \hat{\mathbf{q}}] = (\mathbf{A} \cdot \mathbf{B}) \quad (7.47)$$

(5) Full response functions in $N+\Delta$ space

To show the combined responses from N excitations and Δ excitations, we conventionally define the full isovector spin-vector operators as

$$\tau_\nu \sigma_\mu + \frac{f_{\pi N\Delta}}{f_{\pi NN}} \{T_\nu S_\mu + T_\nu^\dagger S_\mu^\dagger\} \quad (7.48)$$

and present the response functions

$$R_{\nu,X}(q', \omega) = R_{\nu,X}^{NN}(q', \omega) + \frac{f_{\pi N\Delta}}{f_{\pi NN}} \{R_{\nu,X}^{N\Delta}(q', \omega) + R_{\nu,X}^{\Delta N}(q, \omega)\} + \left(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\right)^2 R_{\nu,X}^{\Delta\Delta}(q', \omega) \quad (7.49)$$

with $X = V, L$ or T .

7.5 Ratio of ID_i to R_i

In PWIA the proportionality relations

$$ID_0(\theta, \omega) = K |A_1(q_{\text{cm}})|^2 R_S(q', \omega) \quad (7.50)$$

$$ID_n(\theta, \omega) = K |B_1(q_{\text{cm}})|^2 R_T(q', \omega) \quad (7.51)$$

$$ID_q(\theta, \omega) = K |E_1(q_{\text{cm}})|^2 R_L(q', \omega) \quad (7.52)$$

$$ID_p(\theta, \omega) = K |F_1(q_{\text{cm}})|^2 R_T(q', \omega) \quad (7.53)$$

where K is the kinematical factor, if C -amplitudes are neglected.

Noting these relations, the program presents the ratios

$$\frac{ID_0(\theta, \omega)}{R_S(q', \omega)}, \quad \frac{ID_n(\theta, \omega)}{R_T(q', \omega)}, \quad \frac{ID_q(\theta, \omega)}{R_L(q', \omega)}, \quad \frac{ID_p(\theta, \omega)}{R_T(q', \omega)} \quad (7.54)$$

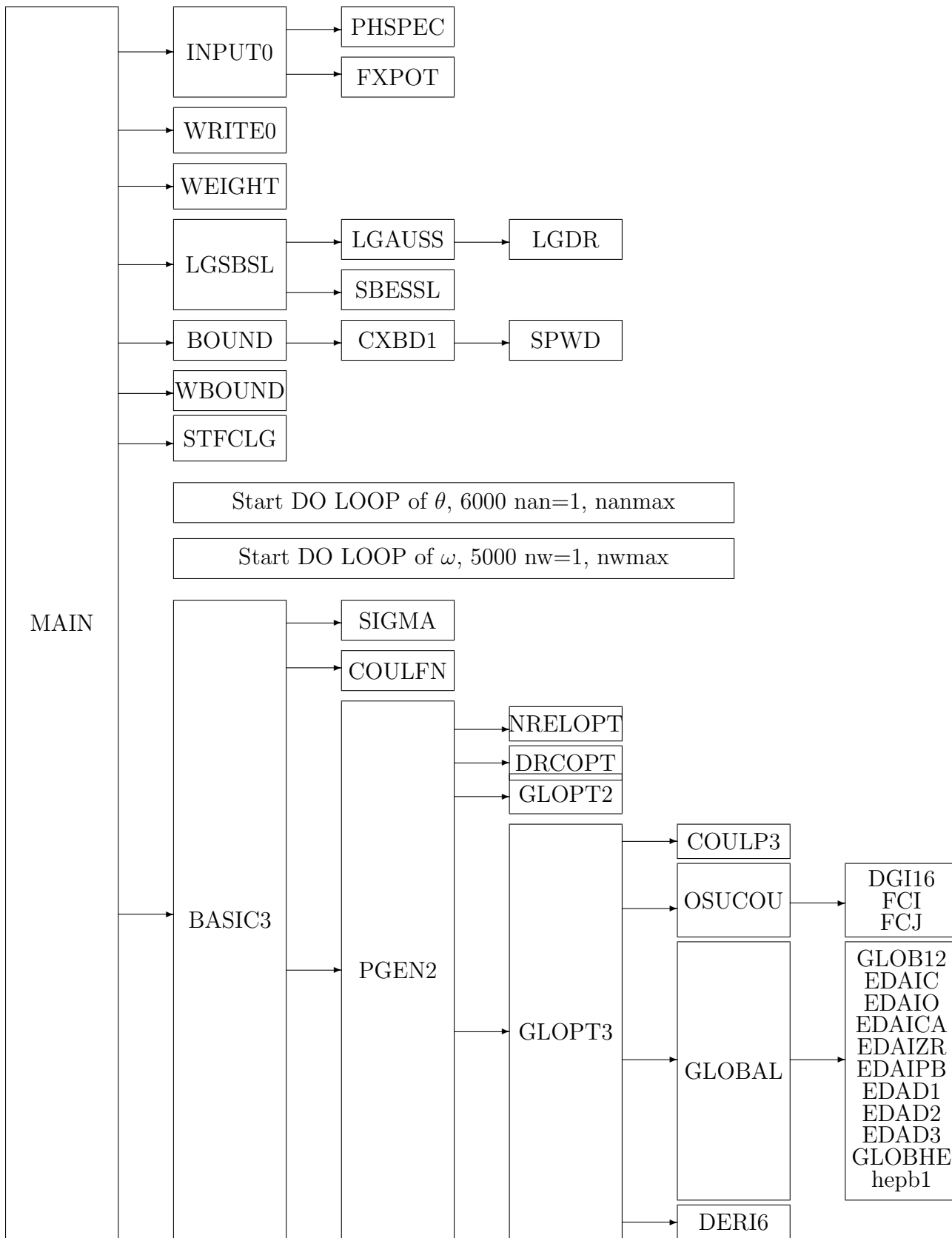
in order to see the effects of distortions.

References

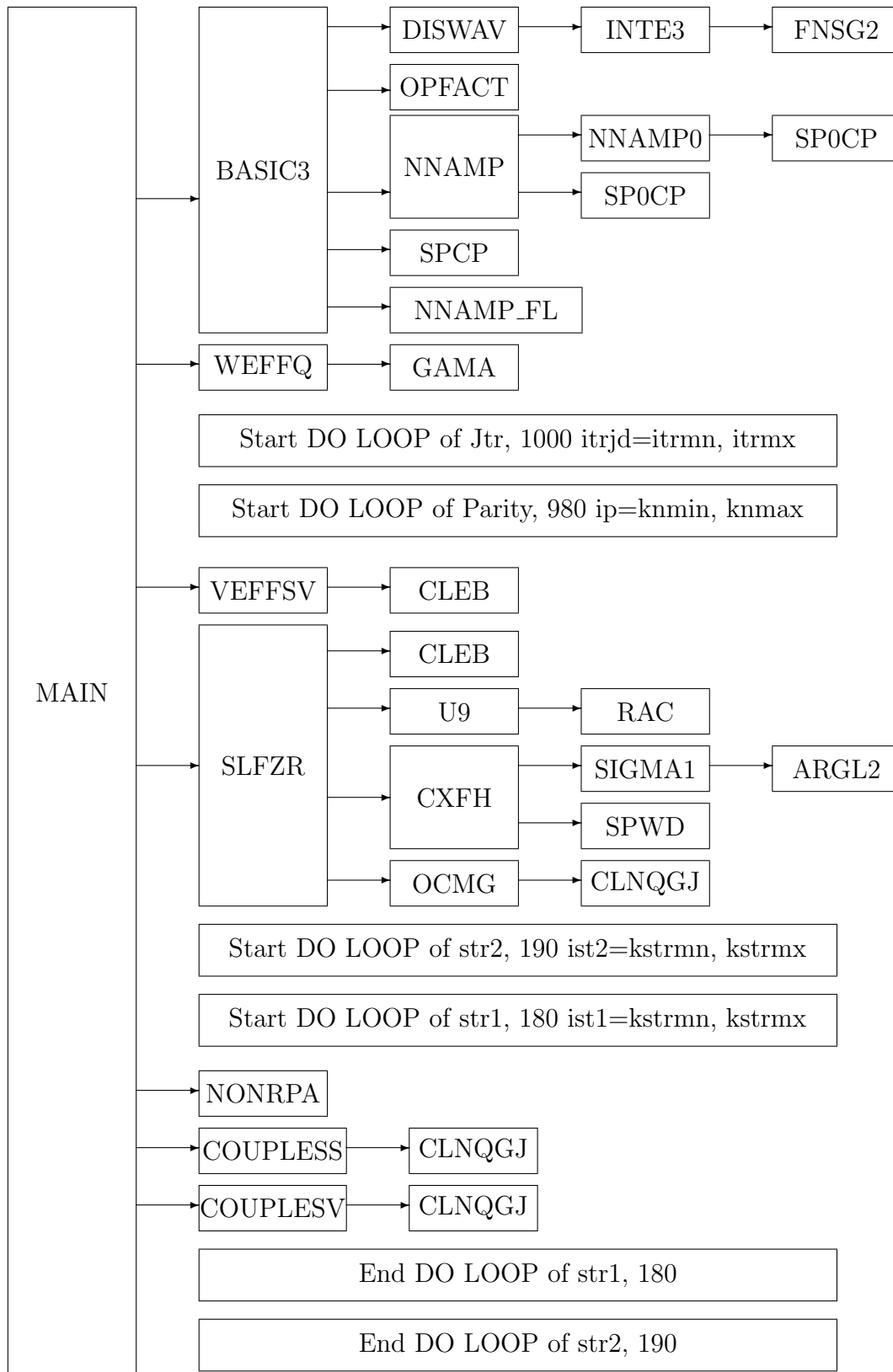
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A Subroutines and Functions

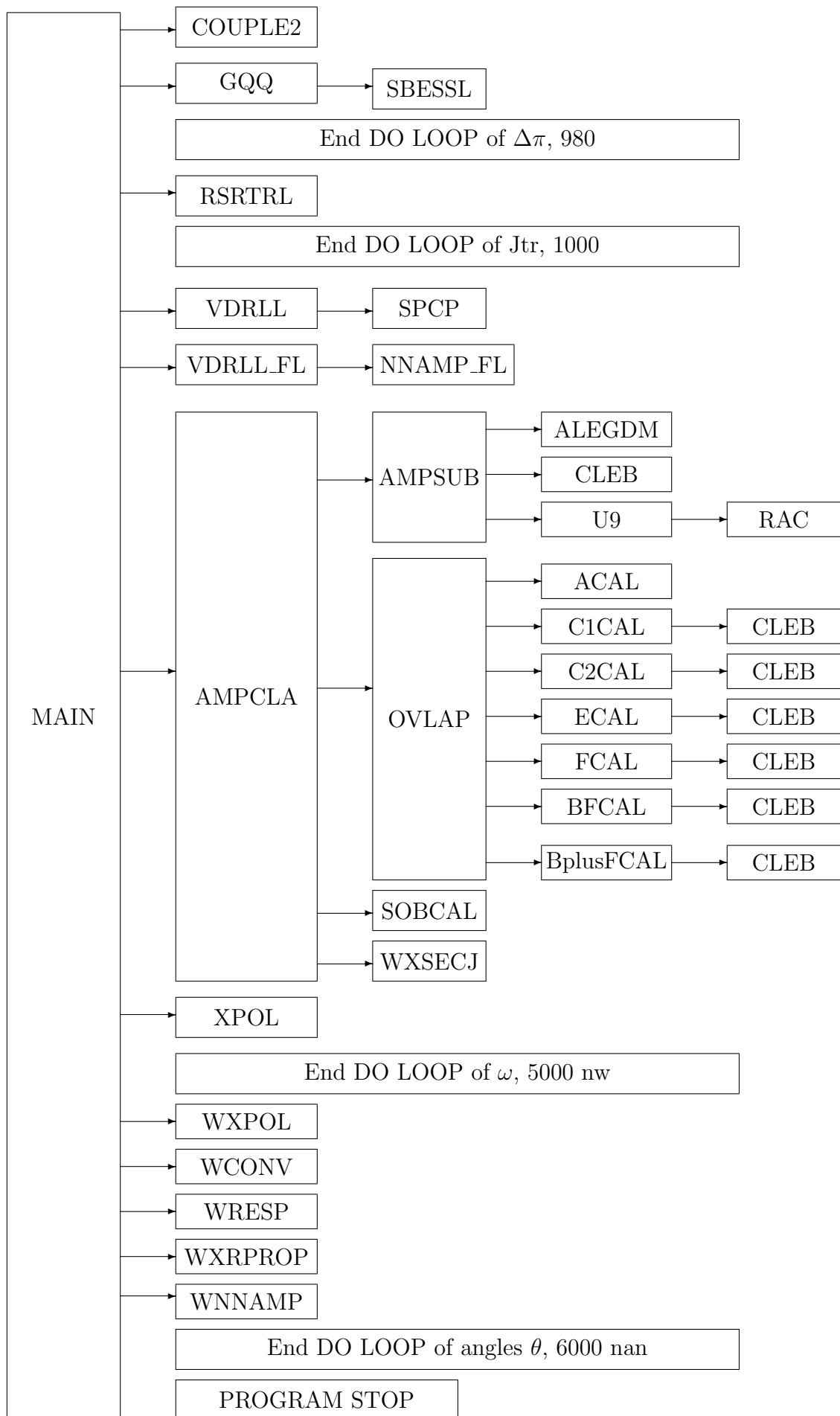
A.1 Structure of subroutines and functions



(continued)



(continued)



A.2 List of subroutines

List of the subprograms in the order of the subroutine structure figures (A.1)

main	main program
input0	Read input data
phspec	Set $ph/\Delta h$ state specification in 'input0'
fxpot	Prefix some potential parameters in 'input0'
write0.	Output the input data
weight	Make table of weights for Simpson integration
lgsbsl	Make table of Legendre-Gauss Integral mesh points, weights and the spherical Bessel function at the mesh points
lgauss	Generate Legendre-Gauss integral mesh points and weights
lgdr	Legendre function
sbessl	Spherical bessel function
stfclg	make table of log of factorials
bound	Set data table of single particle bound state properties and wave functions
cxbd1	Calculate the bound state wave function in the complex potential
spwd	Calculate spreading width of the single particle states
wbound	Output bound state parameters and wave functions
basic3	Prepare kinematic variables, Coulomb wave functions, distorted waves
sigma	Coulomb phase shift
coulfn	Coulomb function
pgen2	Generate the optical potentials
nrelopt	Generate a non-relativistic optical potential
drcopt	Generate the Schrödinger equivalent optical potential from a Dirac phenomenological potential
glopt2	Generate global relativistic optical potential for neutron by Shen et al.
glopt3	Generate the global relativistic optical potential by Hama et al.
coulp3	Subprogram 1 for the global potential given by Hama et al in 'glopt3'
osucou	Subprogram 2 for the global potential given by Hama et al in 'glopt3'
global	Subprogram 3 for the global potential given by Hama et al in 'glopt3'
deri6	Subprogram 4 for the global potential given by Hama et al in 'glopt3'
dgi16	Subprogram a used in 'osucou'
fci	Subprogram b used in 'osucou'
fcj	Subprogram c used in 'osucou'
edad1	Global pot. for C12-PB208 (p,p) 21-1040 MeV (Fit.1)
edad2	Global pot. for C12-PB208 (p,p) 21-1040 MeV (Fit.2)
edad3	Global pot. for C12-PB208 (p,p) 21-1040 MeV (Fit.3)
edaic	Global pot. for C12 (p,p) 29-1040 MeV
edaio	Global pot. for O16 (p,p) 23-1040 MeV
edaica	Global pot. for Ca40 (p,p) 21-1040 MeV
edaizr	Global pot. for Zr90 (p,p) 22-800 MeV
edaipb	Global pot. for Pb208 (p,p) 21-1040 MeV
glob12	Global pot. for Ca40-Pb208 (p,p) 65-1040 MeV Fit.1 & 2
globhe	Global pot. for 4He (p,p) 156-1728 MeV
hepb1	Global pot. for 4HE-PB208 (P,P) 21-1040 MEV
cleb	Clebsh-Gordan coefficient
u9	9j-symbol
rac	Racah coefficient

diswav	Calculate distorted waves
inte3	Solve the Schödinger equation at positive energies with regular and irregular boundary conditions
fnsg2	Fix the boundary condition and normalization for the distorted waves
opfact	Calculate the optimal frame parameters and the Moeller factor for the transformation from the optimal frame to the NN-cm frame
nnamp	Spline the NN t-matrix amplitudes to large q in the off-shell region
nnamp0	Convert the NN t-matrix amplitudes in Bugg's convention to the KMT amplitudes A, C, B, E, F
sp0cp	Initiate the spline function
spcp	Spline operation
nnamp_fl	Convert the Franey-Love NN t-matrix amplitude to the KMT amplitudes
weffq	Calculate isovector spin vector ph effective interaction $W_T(q, \omega), W_L(q, \omega)$
gama	Calculate vertex form factor, $\Gamma(t, m, \Lambda, n) = [(\Lambda^2 - m^2)/(\Lambda^2 - t)]^n$
veffsv	Calculate isovector spin vector ph effective interaction in the r-space
slfzr	Calculate free ph(Δ h) Green's function $G_{\nu's'l_s J}^{(0)}(r, r')$
cxfh	Calculate the regular and singular single particle waves, $f_{lj}(r), h_{lj}(r)$
sigma1	Coulomb phase shift in 'cxfh' $\sigma(l, \eta)$
ARGL2	Function used in 'sigma1'
ocmg	Apply the orthogonality condition to the particle wave functions
chnqgj	Matrix inversion $A^{-1}B$ for complex matrices A and B
nonrpa	Calculate Free ph (Δ h) polarization propagator $\Pi_{\nu's'l_s J}^{(0)}(r, r')$
couples	Calculate the spin scalar polarization propagator in the r-space
couplesv	Calculate the spin vector polarization propagator in the r-space
couple2	Calculate spin vector response functions in RPA $R_{\nu l J}(r, r')$
gqq	Calculate $\Pi_{\nu l J}(q, \omega)$ from $\Pi_{\nu l J}(r, r')$
rsrtrl	Calculate spin scalar and spin vector response functions
vdrll	Calculate driving force from the optimal frame t-matrix in the r-space
vdrll_fl	Calculate driving force from Franey-Love t-matrix in the r-space
ampcla	DWIA calculation for the given $\theta, \omega, J, \Delta\pi, N\Delta$
ampsub	Calculate distorted wave vertex amplitudes
alegdm	Associated Legendre function
ovlap	Calculate $\text{Tr}[TT^\dagger]$
acal	Calculate NN amplitude (A-term)
c1cal	Calculate NN amplitude (C1-term)
c2cal	Calculate NN amplitude (C2-term)
ecal	Calculate NN amplitude (E-term)
fcad	Calculate NN amplitude (F-term)
bfcad	Calculate NN amplitude ((B-F)-term)
bplusfcad	Calculate NN amplitude (A+E+(B+F)/2-term)
sobcal	Calculate cross sections and spin observables for the fixed J and $N\Delta$
wxsecJ	Output cross sections of given J
xpol	Sum up cross sections, P, A_y and D_{ij} over transfer J
wxpol	Output cross sections, P, A_y, D_{ij}
wconv	Output conversion check data (J_{conv})
wresp	Output the response functions, $R_S(\theta, \omega), R_T(\theta, \omega), R_L(\theta, \omega), R_V(\theta, \omega)$
wxrprop	Output the ratio $ID_i(\theta, \omega)/R_i(\theta, \omega)$
wNNamp	Output the absolute square of t_{NN} amplitudes at q_{cm}